

A GENERALIZATION OF THE MOIRE METHOD
WITH APPLICATIONS TO THE THERMAL
STRESS PROBLEM

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LIST OF SYMBOLS

- δ The distance between successive moire fringes.
- p The distance between successive lines in a grid.
- P_i $i = 0, 1, 2, \dots$ points.
- $u(x)$ One dimensional displacement.
- n an integer.
- $u(x,y)$ The component of displacement in the x direction.
- $v(x,y)$ The component of displacement in the y direction.
- u' A fictitious displacement due to pitch mismatch.
- λ An arbitrary constant less than 1.0, also a spatial wave length such as p .
- $U(x,y)$ A total deformation.
- α_n The angular aperture of the lens.
- α A small angle; also an angle defined by the points $M'A'C'$.
- Θ The angle between the normal to a fringe and the coordinate direction perpendicular to the reference grid; A generalized angle in the plane of the model; also the angle defined by the point $MC'R$; also the angle whose cosine is $\cos \frac{1}{2} p \alpha_n$ also an angle equal to $\frac{2\pi u}{p}$.
- ϵ_{ij} The strain tensor.
- ω_{ij} The rotation tensor.
- h A displacement perpendicular to the plane of the model grid.
- D Distance between the plane of the model and the optical center of the lens.

LIST OF SYMBOLS (Continued)

r	A radial distance from the optical axis.
t	The thickness of the model; also the exposure time; also generalized time.
ϵ_{xx}	The component of the strain tensor in the x direction.
ϵ_{yy}	The component of the strain tensor in the y direction.
γ	An arbitrary constant; also the slope of the characteristic curve for films.
V	Michelson's visibility criteria.
I	Intensity of light at a point.
L	One limit to a generalized field at view; or a perfect lens.
A	The other limit to a generalized field of view; or a point source of light.
A'	A point in the image plane corresponding to A.
M	An arbitrary point on the Gaussian sphere; also a magnification ratio.
Σ	A wave front.
π	The image plane.
P	A general point of the image plane.
β, δ'	Coordinate directions in the plane of the lens.
g	Diffraction function of the lens.
θ'	The angle defined by the points $y'A'P'$
ρ	The distance between the points A' and P' .
$J_1(z)$	Bessel function.
$E(y', z')$	The relative illumination of the image plane.

LIST OF SYMBOLS (Continued)

- $I(y,z)$ Luminous flux density of light.
- $O(y,z)$ The luminous flux density of an object.
- $\omega(p,\beta)$ The Fourier transform of $C(y,z)$.
- γ' A spatial frequency.
- ν' A spatial frequency.
- $\Omega(y,\nu)$ The Fourier transform of the image of a point source.
- α_m The same as α_n
- K A constant equal to $2 \frac{D}{\lambda}$; also a parameter which is a fractional multiplier of 1 over one fringe spacing.
- S The areas of integration of the function $\Omega(y',\nu')$.
- a Circular aperture of the lens; also a small constant.
- $F(x,y)$ The generalized transmission function.
- U_o The complex amplitude of the disturbance transmitted by an object.
- U_o^* The complex conjugate of U_o .
- U_i The complex amplitude of the disturbance incident on the object.
- U_i^* The complex conjugate of U_i .
- $F(x)$ The transmission function in the coordinate direction.
- $I(x)$ The one dimensional intensity function in image.
- z Relative coordinate with respect to the reference grid.
- z' Relative coordinate with respect to the image.
- z_l Relative coordinate with respect to the model.
- β A small constant.

LIST OF SYMBOLS (Continued)

- $\bar{P}$ An average pitch in heterogenous displacements.
- I'_0 A fictitious intensity function due to background illumination.
- I'' A fictitious intensity function due to local errors in technique.
- g' An arbitrary multiplication constant.
- ϵ One dimensional strain.
- E_c Exposure, the product of intensity I and the exposure time t .
- D Diffuse density.
- T_n Transmission function of a film.
- O_n Opacity of a film.
- R Electrical resistance.
- E_c Intensity of light falling on a photocell.
- f An arbitrary exponent.
- E_w A voltage from a photocell wheatstone bridge.
- Local displacements.
- F The focal length of a lens.
- $Y(t)$ An arbitrary electric signal that is a function of time.
- $Y(t+\tau)$ The same electric signal $Y(t)$ displaced in time by a constant interval.
- τ The constant time interval.
- U^* Displacement sensitivity - the smallest displacement that can be measured.
- ϵ^* Strain sensitivity - the smallest strain that can be measured.

Abstract of Dissertation Presented to the Graduate Council
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The research discussed in this paper was concerned with the further development of the moire method of experimental stress analysis. Particular emphasis was placed upon achieving complete objectivity in reducing the strain information from the moire pattern. Objectivity, as applied, implies a method free of measurements made by the operator's sensory perceptors. The objective method was applied to several tests including a simple tension sample, a ring under diametral compression, and three applications to the thermal stress problem of a ring under an axisymmetric temperature distribution. Maximum temperatures achieved were 1,580°F. The results indicate that new levels of sensitivity and accuracy were achieved though coarse gratings were used.

The moire method used previous to this research was discussed. A new procedure was outlined and analysed. A previously unknown error in the moire method due to background illumination variation was found and a solution was proposed to eliminate this error by the use of special hand analysis technique. A photo-reading device was designed and built. A computer program was developed to reduce the moire data. Techniques for using the moire method in thermal stress problems at moderately high temperatures were devised.

The conclusions drawn from this research were:

1. That the moire method of experimental stress analysis can be made completely objective such that strains at a point on a deformed body can be ascertained for the first time.

2. That the accuracy and sensitivity of the method can be improved by a magnitude of order of ten by the use of a photo-reader, a method of data reduction and a computer over the existing hand methods of analysis.

3. That the moire method of stress analysis is applicable to two-dimensional thermal stress problems to temperatures of 1,700°F.

CHAPTER I

INTRODUCTION TO THE PROBLEM

The research reported in this paper covers the application of the moire method to a problem of thermal stress. An attempt has been made to increase the objectivity, accuracy, and sensitivity of the moire method. Certain improvements in techniques, construction of a precision camera, a photo reader for data recovery, an optical displacement law, and a computer program have brought the moire method into the ranks of the most useful tools available for experimental stress analysis. The application of these principles in a completely objective fashion to the thermal stress problem of a stainless steel ring, which is heated on its outer circumference while cooled on its inner boundary, has shown the merit of the system. Tests were conducted with maximum temperatures ranging from 350°F to 1,700°F.

In the remainder of this paper, when broad concepts which can best be understood as full generalities are involved, tensor notation will be used. When more specific discussion is required, normal mathematical notation will be used.

1.1 Statement of the Problem

The problem of this research is to develop the techniques of thermal stress analysis and the techniques of data reduction of the moire method of experimental stress analysis to the highest level of objectivity consistent with the requirements of the stress analyst and apply these techniques to the thermal stress problem of a ring under axisymmetric radial temperature distribution.

1.2 Statement of the Need for this Research

The need for the study of thermal stress problems is self-explicit in these times of high energy utilization. The experimental study of thermal stress in solids has been hampered by the lack of suitable experimental stress analysis methods.

Through work sponsored by the National Science Foundation to develop a method of using moire fringe patterns to examine thermal stress fields, the author has realized that an important secondary need exists. As is the case many times in research, a solution to the secondary problem is as beneficial to mankind as is the answer to the original question, the secondary need in this case being to develop the objectivity of the moire method. The benefit to mankind from this research is that

it will make available to all an idealized method of two-dimension stress analysis for the study of stress field problems, regardless of the type of boundary conditions. Further exploration and development of this need involves a review of current and past stress analysis methods in an objective nondetailed fashion.

The history of modern experimental stress analysis methods began in 1913 when Meanager and, separately, Coker made an engineering tool of photoelasticity (1).* Since that time many methods have been developed, flourished, and vanished with the originator. Methods that have found acceptance and use include: mechanical extensometers, including optical refinements; electrical resistance strain gages; pneumatic strain gages; X-rays, photoelasticity; acoustic methods; brittle lacquer coatings; photoelastic coatings; grid methods; and most recently, the moire method.

In order to establish a criterion for the evaluation of these methods, an ideal experimental method may be considered. The ideal experimental stress analysis method would be able:

*Whole numbers in parentheses refer to items in the List of References at the end of this dissertation. Numbers containing decimals refer to equations.

- (1) To accommodate one, two, and three-dimensional stress field problems.
- (2) To be applicable to elastic, elastoplastic, and plastic problems.
- (3) To accommodate an infinite variety of boundary conditions.
- (4) To possess an objectivity in use and in results obtained which is free from human measurements other than normal diligence to detail in technique.
- (5) To yield reproducible information related to stress at every point in a discrete, unambiguous form within defined limits of sensitivity and accuracy.

Here sensitivity is defined as the smallest stress related unit that can be detected by the method. Accuracy is defined to mean more than mere precision. Precision implies only reproducibility while accuracy implies freedom from systematic error which may reproduce.

When the contemporary existing methods of experimental stress analysis are compared to the ideal, all are found to possess some defects. Mechanical-optical extensometers and electric strain gages are essentially one-dimensional strain field measuring devices. The results obtained are average strains over a finite gage

length. These methods will accommodate a variety of problem boundary conditions, including thermal boundary conditions, provided the resulting strain field is uniform and one-dimensional. Thus, these methods are ideal for a uniaxial test of material properties.

A multiplicity of electric strain gages can be applied to a model to obtain an approximation of a two-dimensional strain field. With thermal boundary conditions, the complications induced by the need to compensate thermally each strain gage may be excessive. The electric strain gage used in a two-dimensional field has departed from the ideal method. These limitations notwithstanding, electric strain gages represent the most widely used method of stress analysis, principally because of the inherent objectivity of an electric signal.

The domain of two-dimensional stress analysis is dominated by photoelasticity, photoelastic coating, and recently, moire. Of these, photoelasticity has been the most widely used method. In the photoelastic method, information is obtained relating the difference in magnitude of the average principal stresses through the thickness of the photoelastic model as well as the directions of these average principal stresses. Outstanding results have been achieved by the photoelastic method under

varied boundary conditions by many workers. Generally, the reduction of data from this method is a highly subjective procedure involving great operator skill and intuition in the performance of measurements that are a necessary part of the method. The complete modeling of a problem having thermal boundary conditions is not possible using photoelastic materials. Also, the viscoelastic properties of most photoelastic materials complicate the application to the method to problems with dynamic boundary conditions.

Photoelastic coatings are relatively new to the stress analysis field but indications are that the method suffers the same subjective forms of analysis that accepted photoelastic techniques possess.

That objectivity in the method of stress analysis is a sufficient goal in itself is emphasized by the most prevalent use of photoelasticity, that is, the determination of stress concentration factors in two-dimensional stress fields. This determination is made by simply counting discrete integer fringe orders and forming a ratio, a highly objective procedure.

The moire method, prior to this research, yielded information about the relative displacements of points on the surface of a model. The information

specifically obtained was loci of points of equal displacements, each displacement being of magnitude equal to the pitch of a reference grid. Again, in all generality, the reduction of data for the method is a highly subjective procedure because of critical measurements that must be performed by the operator. Many fine works have been accomplished with this method through the display of care, diligence, skill, and intuition of a few people. The principal strength of the method is in the ability to accept a variety of types of boundary conditions with engineering materials in two-dimensional problems.

The extension of an existing method to problems of three-dimensional character is more a problem of modeling techniques than an inherent capability of the method. For instance, electric strain gages are regularly imbedded in concrete structures to approximate 3-D stress fields. Photoelasticity and moire have been used in 3-D problems using "frozen stress" techniques or transparent models, the problem being principally selected to use the two-dimensional character of the method.

The necessarily brief discussion of the status of experimental stress analysis methods has been

included here to point up the need for a highly objective method of two-dimensional stress analysis. Accuracy and sensitivity can only be discussed in rational terms after objectivity has been attained in some method.

It is recognized that no method can be completely isolated from the influence of the operator. Normal diligence, care, and skill are required in every phase of every method for successful application of that method. The objectivity sought for in this thesis is the freedom of the method from measurements performed by the operator using his sensory perceptrors, whether sight, sound, pressure, pain, or smell. It is known that sensory perception varies between individuals and between times and environments for any individual. It is recognized that interpretation of results is the rightful domain of the stress analyst and that collection and reduction of data should be as free from personal interpretation as measurements can be.

The moire method of stress analysis has the ability to be applied to one, two, and three-dimensional problems in the elastic, elastoplastic and plastic domains by virtue of its geometric character. The ability to accept static, dynamic and thermal boundary

conditions applied to all engineering materials makes the moire method an ideal vehicle with which to attempt complete systematism.

1.3 A Discussion of the Thermal Aspect of the Problem

The objectives of the National Science Foundation research contract were specific, that is, to develop the techniques of using the moire method in thermal stress problems and to investigate a problem of thermoelasticity and thermoplasticity. The work presented in this paper is an extension of previous work (2) by the author to develop the method to be useful to high temperatures. The full implications and results of the thermal stress problem will be reported later.

To develop the method, a static thermoelastic problem was selected. The thermoelastic problem can be as defined by Boley and Weiner (3).

Given a regular region of space $D + B$ with boundary B , then there exists at most one set of single valued functions $\sigma_{ij}(P, t)$ and $\epsilon_{ij}(P, t)$ of class C^1 , $u_i(P, t)$ and $T(P, t)$ of class C^2 for $P(x_1, x_2, x_3)$ in $D + B$ for $t \geq 0$, which satisfy the equations for P in D , $t \geq 0$.

$$K T_{,mm} = \rho C_E \dot{T} + m T_0 \dot{\epsilon}_{KK} \quad (1.3.1)$$

$$\sigma_{ij,j} = \rho \ddot{u}_i \quad (1.3.2)$$

the following equations for P in D + B, $t \geq 0$

$$\epsilon_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}) \quad (1.3.3)$$

$$\sigma_{ij,j} = \delta_{ij} \lambda \epsilon_{KK} + 2\mu \epsilon_{ij} - \delta_{ij} m (T - T_0) \quad (1.3.4)$$

the following equations for P on B, $t > 0$

$$T = F^{(1)}(P, t) \quad (1.3.5)$$

$$u_i = G_i^{(1)}(P, t) \quad (1.3.6)$$

the following equation for P in D, $t = 0$

$$T = F^{(2)}(P) \quad (1.3.7)$$

$$u_i = G_i^{(2)}(P) \quad (1.3.8)$$

$$\dot{u}_i = G_i(P) \quad (1.3.9)$$

where the quantities K , C_E , $3\lambda + 2\mu$, μm , and T_0 are positive. This is a complete statement of the coupled unique thermoelastic problem, and proof is given in the same text.

The mathematical difficulties of a theoretical solution to the coupled equations (1.3.1) and (1.3.2)

are formidable. The usual methods include considering the uncoupled equation; that is, $\dot{\epsilon}_{\kappa\kappa} \approx 0$, or the further simplification of considering $\ddot{u}_i \approx 0$.

With an experimental method, the initial condition functions $F^{(2)}(P)$, $G^{(2)}(P)$, and $G^{(3)}(P)$ can be measured. The boundary condition function $F^{(n)}(P, t)$ and $G_i^{(n)}(P, t)$ can be established within the limitations of the method used. The moire method has no limitations in this respect. The measurement of μ_i and the use of (1.3.3) and (1.3.4) constitute a solution to the most general thermal elastic problem. The equation (1.3.4) for an isotropic linearly elastic solid is known as the Duhamel Neumann stress-strain relation. The importance of this law in any experimental method implies the need to discuss the formulation of this law.

Starting with the equation which expresses the conservation of energy for an elastic solid,

$$\sigma_{ij,j} \dot{\epsilon}_{ij} + \rho T \dot{\eta} = \rho \dot{\mathcal{E}} \quad (1.3.10)$$

where $\mathcal{E}$, η , and T are the specific internal energy, entropy density, and absolute temperature, respectively. The dot represents the usual time rate.

A free energy function is defined:

$$\phi(\epsilon_{ij}, T) = \mathcal{E}(\epsilon_{ij}, T) - T\eta(\epsilon_{ij}, T) \quad (1.3.11)$$

the substitution of (1.3.11) in (1.3.10) results in

$$\left(\sigma_{ij} - \rho \frac{\partial \phi}{\partial \epsilon_{ij}} \right) \dot{\epsilon}_{ij} - \rho \left(\eta + \frac{\partial \phi}{\partial T} \right) = 0 \quad (1.3.12)$$

which implies

$$\sigma_{ij} = \rho \frac{\partial \phi}{\partial \epsilon_{ij}} \quad (1.3.13)$$

Setting

$$\phi(\epsilon_{ij}, T) = \phi(I_\epsilon, II_\epsilon, III_\epsilon, T) \quad (1.3.14)$$

where I_ϵ , II_ϵ , and III_ϵ are the invariants of the isotropic strain tensor.

It follows from

$$\sigma_{ij} = \rho \left(\frac{\partial \phi}{\partial I_\epsilon} \frac{\partial I_\epsilon}{\partial \epsilon_{ij}} + \frac{\partial \phi}{\partial II_\epsilon} \frac{\partial II_\epsilon}{\partial \epsilon_{ij}} + \frac{\partial \phi}{\partial III_\epsilon} \frac{\partial III_\epsilon}{\partial \epsilon_{ij}} \right) \quad (1.3.15)$$

Since

$$I_\epsilon = \epsilon_{ii} \rightarrow \frac{\partial I_\epsilon}{\partial \epsilon_{ij}} = \delta_{ij} \quad (1.3.16)$$

$$II_\epsilon = \frac{1}{2!} \delta_{lm}^{ij} \epsilon_{ik} \epsilon_{jm} \rightarrow \frac{\partial II_\epsilon}{\partial \epsilon_{ij}} = \delta_{ij} I_\epsilon - \epsilon_{ij} \quad (1.3.17)$$

$$III_\epsilon = \frac{1}{3!} \delta_{lmn}^{ijk} \epsilon_{il} \epsilon_{jm} \epsilon_{kn} \rightarrow \frac{\partial III_\epsilon}{\partial \epsilon_{ij}} = \epsilon_{ik} \epsilon_{jk} - \epsilon_{ij} I_\epsilon + \delta_{ij} II_\epsilon \quad (1.3.18)$$

It is assumed a stress-free reference state

exists at a temperature T_0 . Then defining T' :

$$T' = \frac{T - T_0}{T_0} \quad (1.3.19)$$

The free energy function is expanded in a power series from the stress-free condition.

$$\begin{aligned} \Phi(I_\epsilon, II_\epsilon, III_\epsilon, T') = \frac{1}{\rho} & (a_0 + a_1 I_\epsilon + a_2 II_\epsilon + a_3 III_\epsilon \\ & + a_4 T' + a_5 I_\epsilon^2 + a_6 II_\epsilon^2 + a_7 III_\epsilon^2 + a_8 T' I_\epsilon + \dots) \end{aligned} \quad (1.3.20)$$

where the α_i 's are constants. A linear theory restricts the terms in higher order products of ϵ_{ij} and T' .

Thus from (1.3.13)

$$\sigma_{ij} = \rho \left[\frac{1}{\rho} (a_0 + a_1 II_\epsilon + \dots) \right] \quad (1.3.21)$$

$$\begin{aligned} \sigma_{ij} = a_1 \delta_{ij} + a_2 (\delta_{ij} \epsilon_{KK} - \epsilon_{ij}) + 2a_5 \delta_{ij} \epsilon_{KK} \\ + a_4 \delta_{ij} T' \end{aligned} \quad (1.3.22)$$

$$\sigma_{ij} = (2a_5 + a_2) \delta_{ij} \epsilon_{KK} - a_2 \epsilon_{ij} + a_4 \delta_{ij} T' \quad (1.3.23)$$

Considering the cases when $T' = 0$ are pure thermal dilatations,

$$\begin{aligned} a_2 &= -2\mu \\ a_5 &= \frac{1}{2} (\lambda + 2\mu) \\ a_4 &= (-3\lambda + 2\mu) \alpha T_0 \end{aligned} \quad (1.3.24)$$

and

$$\sigma_{ij} = \lambda \delta_{ij} \epsilon_{KK} + 2\mu \epsilon_{ij} - (3\lambda + 2\mu) \alpha (T - T_0) \delta_{ij} \quad (1.3.25)$$

The implications of this statement are that:

(1) The statement is useful to such values of $T' = \frac{T - T_0}{T_0}$ that higher products of ϵ_{ij} and T' be neglected in the calculation of stresses.

(2) The free energy function being expanded from a stress-free condition in T' does not limit the coefficient of the power series from being a function of the absolute temperature T .

Comparison of actual strains from an experimental program with calculated strains from a theoretical solution using the Duhamel law should show the range of validity of the law. As previously stated, this has been accomplished in a thermoelastic static problem of a ring subjected to a radial temperature gradient. Here

$$T_{MAX} = 500^{\circ}F \quad T_0 = 80^{\circ}F \quad (1.3.26)$$

or

$$T_{MAX} = 960^{\circ}R \quad T_0 = 540^{\circ}R$$

and for a maximum temperature point

$$\frac{T - T_0}{T_0} = \frac{420}{540} = 0.78 \quad (1.3.27)$$

Good agreement was observed between calculated and actual strains in this previous case.

1.4 Review of Literature

The solutions of certain theoretical problems have established the foundations for recent analytical solutions to thermal stress problems. Danilovskaia (4) has studied the effect of inertia in the transient thermoelastic problem. The problem studied was an elastic half space subjected to a sudden temperature change along its entire plane boundary. A second paper by the same author (5) expanded the problem to include convective heat transfer. Mura (6) obtained the same solution independently. Steinberg and Charkravorty (7) generalized the same problem to include a gradually applied heat load.

Boley and Tolins (8) considered the effect of thermal coupling in the dynamic problem. Muki and Breuer (9) independently solved the coupled dynamic thermoelastic problem for the half space. Both papers show that because of the velocity of propagation of thermal dilatation waves and the necessity of continuity of displacements, the stress discontinuities are greatly reduced.

The results of these classic analyses have led to the conclusion that for massive bodies, consideration of the uncoupled quasistatic thermal problem is sufficient.

Boley (10, 11) has carried out a similar analysis for slender structures, including beams and shells. The conclusions were similar except for the case of very thin shells.

While these conclusions are accepted and used, very little experimental confirmation exists.

The earliest experimental approach to thermal stress was included in a paper by Weibel (12) in 1938. Weibel used Biot's analogy (13, 14), in which a photoelastic model is subjected to a Volterra type discontinuity. No thermal boundary conditions are involved. Biot's analogy has been used by Barriage and Durelli (15) and also by Ordahl and Williams (16) for the study of thermal stress, principally in solid propellant grains. Daniel and Durelli (17) have compared the results of the mechanical analogy and direct thermal loading in the case of bonded propellant grains. Reichner (18) has studied the effect of notch spacing and notch radius on factors of stress concentration under steady thermal loading. Reichner applied heat in his technique.

Leven and Sampson (19) used the photoelastic technique of "frozen stress" to construct a similar model whose elements contain strains proportioned to the free thermal expansion of the prototype element.

• Heywood (20) and Leaf (21) introduced photoelastic methods of studying the residual stresses due to thermal contraction in complicated parts.

By defining scaling factors for the use of photoelasticity in thermal stress problems, Gerard and Gilbert (22) have coined a new word in stress analysis, "photothermoelasticity." This method is an approach to the study of transient thermal problems. Trampusch and Gerard (23) investigated the properties of materials suitable for photothermoelasticity. Some use of the method has been made in the study of cemented I beam structures (24) and three-dimensional bodies (25). By using polariscope elements cast within a model, thermal stress in cylinders has been investigated (25).

Birefringent coatings on photoelastic coatings have been used by Convair Astronautics to study thermal stress in space vehicles (26). Zandman (27) reports the temperature limit for birefringent coatings is 500°F for short periods and 300°F for long periods. Duffey and Milonas (28) reported on the effect of thickness of birefringent coatings.

Concerning the moire method, the earliest reference to the method was made by Tollenaar (29) in 1945 and is considered to be the source of the now

obsolete "geometric approach." This method was applied to strain determinations for the first time in 1952 by Kaczer and Krowpa (30). Similar approaches are presented in other papers (31, 32, 33, 34). Bromley (35) obtained results similar to previously cited results (29, 30) by using tensor notation.

In 1948, Weller and Sheppard (36) presented a slightly varied approach. In 1954, Dantu (37) introduced the relation between the fringes of the moire pattern and the components of displacement. Thiry and Dantu (38) expanded the theory of the moire method. Diruy (39) applied the method to various problems of aeronautical concern. This work describes the first use of moire in strain gage techniques. Sciammarella (40) corrected Dantu's interpretation of the moire method and extended the method to finite deformations.

Dantu (41) extended the use of moire to the problem of residual thermal stresses in two thin pieces of aluminum joined by welding. In this work he was unable to photograph the heated model. He used a grid containing 625 lines per inch.

Loof (42) used the moire method and the slab analogy to determine the distribution of moments and

thermal stresses in cross sections of tunnels and to determine thermal stresses in multiply connected slices (43).

Sciammarella (40) has shown the practicality of using the moire method to thermal stress problems by demonstrating the formation of fringes in low modulus material subjected to cooling.

The first application of the moire method using grids of 1,000 lines per inch to a thermal stress problem was the work completed by the author for a Master of Science thesis and presented to the Society for Experimental Stress Analysis in the fall meeting of 1963 by Sciammarella and Ross (2).

Theocaris (44) has applied the moire method to the study of elasto-plastic behavior at elevated temperature of a tension sample. Grids of 500 lines per inch were used.

While the literature of these associated fields of facsimile reproduction and television were examined, no specific information was obtained because the limit of resolution in these fields is much greater than can be tolerated in a system to analyse photographs for stress analysis.

CHAPTER II

FOUNDATIONS OF THE EXPERIMENTAL METHOD

2.1 Fundamentals of the Moire Method Prior to this Research

The moire method receives its name from the similarity to the optical patterns observed in the method and those of particular forms of watered silk bearing the same name.

Essentially, the moire method consists of the use of two grids composed of finely spaced lines. The width of the line is usually the same as the width of the space separating the lines. Fine spacing of 300 to 1,000 lines per inch has been used in the moire method. One set of lines (usually ruled in orthogonal directions) is applied directly to an undeformed model by suitable techniques. The lines on the model are superimposed either mechanically or optically upon the reference grid (usually ruled in one direction only). The superposition can be accomplished by directly placing the transparent reference grid upon the ruled model surface, by imposing the image of the model on the reference grid in an optical system such as a camera, or by merely photographing the model and applying the

reference grid to the resultant photographic negative of the model.

If the grids are of identical spacing, called pitch, and are aligned perfectly, no fringe pattern is visible. Upon loading the model by means of a device suitable to the boundary conditions of the stress analysis problem being studied, fringes appear due to the mechanical interference between the deformed model grid and the unstrained reference grid.

The mechanism of fringe formation can be understood by considering a one-dimensional case. Suppose that initially both grids are identical and exactly superimposed. One pattern, that of the lines themselves, would be visible. Now, if one grid is extended uniformly, a condition is obtained in which at a point p_0 the lines will coincide exactly, at a point p_1 displaced a distance δ , the lines will coincide exactly again. At all other points the lines of the two grids do not coincide. Indeed, at some point p_x between p_0 and p_1 a line of the deformed grid will be exactly over the space of the undeformed grid if the displacement is uniform. Since the lines and the space have unequal transmission or reflection functions for incident light, a moire fringe pattern is visible. The centers of the light fringes coincide with points p_0 and p_1 while the center of the dark fringe will be found

at p_x . This phenomenon can occur also under uniform compression of the model grid. Now if the requirement for uniform displacement is dropped, it is obvious that the only effect is that p_x will be located at some point other than the center point between p_0 and p_1 .

The implications of this description are that, if p_0 is considered a reference, a point finally located at p_1 has experienced a displacement of one pitch of the reference grid. In a succeeding fringe, a point p_2 , located at the next registry of the two sets of lines, has experienced a relative displacement with respect to point p_0 of two pitches, or $2p$. A point p_n in the n^{th} register of the two grids is a point whose relative displacement with respect to p_0 is np .

If a coordinate direction x is taken perpendicular to the line of the reference grid, then

$$u(x) = np \quad (2.1.1)$$

is a description of the relative displacement of points in the moire fringe patterns for a one-dimensional displacement.

The direction orthogonal to x is denoted y in the usual sense. The p_k are general points of the model grid whose displacements are functions of spatial coordinates (x, y) and usually defined by their scalar

components u_i in the coordinate direction. The component in the direction x is u and in the direction y is denoted v . Then in a general case

$$u(x, y) = np \quad (2.1.2)$$

is the equation of the relative displacement of points in the x direction of the moire fringes in the two-dimensional case.

A rotation of the reference grid through an angle of $\frac{\pi}{2}$ yields a set of fringes characterized by

$$v(x, y) = np \quad (2.1.3)$$

Equations (2.1.2) and (2.1.3) are the fundamental relationships in the moire method prior to the research.

It will be noted that these are the equations of a family of curves known as contour lines in the function $u(x, y)$ and $v(x, y)$. The fringe pattern is the projection of these contour lines on the plane of the base of the surfaces $u(x, y)$ and $v(x, y)$. The fringes are thus loci of points of equal relative displacements, the relative displacements being np .

Fringes may also be formed by using grids of different pitch on the model and in the reference grid. With the model in the undeformed condition a set of fringes will be visible. These fringes represent a fictitious displacement u' .

If the pitch of the model grid is p' and

$$p' = p(1 - \lambda), \quad \lambda \ll 1 \quad (2.1.4)$$

then

$$u' = -\lambda x \quad (2.1.5)$$

an initial moire fringe pattern will exist

$$u' = np \quad (2.1.6)$$

After deformation the fringe will represent the total displacement $U(x, y)$. Where u is the deformation,

$$U(x, y) = u(x, y) + u'(x, y) = u - \lambda x \quad (2.1.7)$$

A moire pattern may also be formed by grids of equal pitch without the deformation if one grid is rotated with respect to the other by a small angle. To illustrate this, consider moire of u $u(x, y)$ (reference grid is vertical), and the lines of the model grid are rotated a small angle α with respect to the lines of the reference grid. Points on the model experience a rigid body rotation, which result in a moire pattern corresponding to a fictitious displacement u'' ,

$$u'' = \alpha y \quad (2.1.8)$$

After deformation the total displacement U determines the moire pattern.

$$U(x, y) = u + u'' = u + \alpha y \quad (2.1.9)$$

Obviously these conditions can exist together and

$$U = u + u' + u'' = u - \lambda x + \alpha y \quad (2.1.10)$$

For a general case, the angle that the normal to a fringe makes with the coordinate axis perpendicular to the lines of the reference grid can be expressed

$$\tan \theta = \frac{\frac{\partial U}{\partial y}}{\frac{\partial U}{\partial x}} = \frac{\frac{\partial u}{\partial y} + \alpha}{\frac{\partial u}{\partial x} - \lambda} \quad (2.1.11)$$

thus

$$\theta = \theta(\alpha, \lambda) \quad (2.1.12)$$

$$d(\tan \theta) = \frac{\partial \tan \theta}{\partial \alpha} d\alpha + \frac{\partial \tan \theta}{\partial \lambda} d\lambda \quad (2.1.13)$$

This can be useful in two ways. First, assuming a deformation of unknown sign with $\lambda = 0$, $d\lambda = 0$

$$\text{then } d(\tan \theta) = \frac{1}{\frac{\partial u}{\partial x}} d\alpha \quad (2.1.14)$$

For arbitrary ~~dx~~ change in angle of the normal to the fringe can be observed. If the rotation of the fringes is in the same sense as the rotation of α , then it is known the sign of the strain must be positive or traction. Conversely, opposite rotation of fringes yields a compression sign to the strain.

Second, if $u(x, y) = 0$, $d\alpha = 0$,

$$d(\tan \theta) = -\frac{\alpha}{\lambda^2} d\lambda \quad (2.1.15)$$

then the necessary condition for $d(\tan \theta) = 0$ for arbitrary $d\lambda$ is $\alpha = 0$. This fact can be useful for adjusting α .

The moire fringe patterns are related in this manner to the displacement field according to the previous development of the method.

The smallest stress related units measured by the moire method are the components of the strain tensor

$$\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}, \text{ or } \frac{\partial u_i}{\partial x_j}, \text{ or } u_{i,j} \quad (2.1.16)$$

These strains and rotations are then defined by the usual relations (small strain theory)

$$\epsilon_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}) \quad (2.1.17)$$

$$\omega_{ij} = \frac{1}{2} (u_{i,j} - u_{j,i}) \quad (2.1.18)$$

In the application of the moire method prior to this research the curves of $u(x, \kappa), v(1, y)$ with $y = \kappa$ and $\kappa = 1$ were reconstructed using np as the ordinate and fringe location as the abscissa. Graphic differentiation of these curves then yields the strain components.

Efforts by Dantu (37, 41) Sciammarella (40), and recently by Theocaris (44) and the author (2) have resulted in techniques which make the model preparation through the photographs of the resulting moire pattern highly objective, subject only to reasonable skill and diligence. It is in the analysis of the photographs and obtaining the strain components that the subjective element of the moire method enters.

2.2 Further Consideration of the Moire Method Prior to this Research

Sciammarella (40) has given a description of the errors that may be involved in the use of the moire method from the selection of a grid to the image in the plane of the photographic negative. With proper care these errors may be inconsequential in magnitude. The errors which arise from systematic and progressive errors in the original master grid can be eliminated by carefully checking the moire pattern produced by the rotation of a copy and the original grid and rejecting imperfect grids.

The effect of non-coincidence of the planes of the model grid and the reference grid can be shown as follows. A fictitious displacement u' exists at each point caused by the image forming light rays from the optical center of a lens at a height D from the reference phase.

Considering a point p a distance r from the origin on the x axis, one obtains:

$$\frac{h}{D} = \frac{u'}{r} \quad (2.2.1)$$

or at any other point

$$u' = \frac{hr}{D} \sin \theta \quad (2.2.2)$$

with

$$r = \sqrt{x^2 + y^2} \quad \sin \theta = \frac{x}{\sqrt{x^2 + y^2}} \quad (2.2.3)$$

$$u' = \frac{hx}{D} = np \quad (2.2.4)$$

Thus if h is a constant, the fringes of moire will be parallel straight lines. In general, $h = h(x, y)$

$$u' = \frac{h(x, y)x}{D} = np \quad (2.2.5)$$

This corresponds to an initial pattern to be subtracted from $U(x, y)$, the total displacement to yield $u(x, y)$, the displacement due to distortion. Thus, if an initial gap occurs between the plane of the reference grid and model grid, either by mechanical superposition; by the image forming process; or by the superposition of negatives, it can result in no error by using the difference technique so long as the gap is repeated in the strained state of the model.

There are gaps which may occur as the result of loading due to a displacement in the z direction, normal to x and y . For these

$$h(x, y) = w(x, y) = -\frac{t\nu}{z} (\epsilon_{xx} + \epsilon_{yy}) \quad (2.2.6)$$

where t is the thickness of the model. Then

$$\frac{\partial u'}{\partial x} = \frac{\partial w(x, y)}{\partial x} \cdot \frac{x}{D} + w(x, y) \frac{1}{D} \quad (2.2.7)$$

Considering

$$\gamma |\epsilon_{xx}| = |\epsilon_{yy}| \quad (2.2.8)$$

$$\left| \frac{\epsilon_{xx'}}{\epsilon_{xx}} \right| = \frac{\frac{\partial u'}{\partial x}}{\frac{\partial u}{\partial x}} = \frac{t\nu}{zD} \quad 1 + \gamma \quad (2.2.9)$$

with $t = 1.0$ inch, $\nu = \frac{1}{2}$, $\gamma = 1$, $D = 24$ inches

the maximum variation due to the imposed loading is

$$\left| \frac{\epsilon'_{xx}}{\epsilon_{xx}} \right| = \frac{1.0}{3 \times 24} = .013 \text{ or } 1.3\% \quad (2.2.10)$$

With these demonstrations, it has been shown that it is possible to achieve an accuracy of approximately 99 per cent in the moire method in the transformation of strains to displacement to points on a photographic negative by use of the differential method.

Consider now the subjective portion of moire data analysis. The assumption is made that the image forming lens is perfect and apochromatic or that monochromatic light has been used in forming the negative. The operator must make two measurements from the photographic negative. One, he must determine precisely the most dense region of the dark fringe, and two, he must measure the coordinate location of this most dense point. It has been found that a circle having a diameter of 1/100 inch appears to the normal eye as a point when observed at the normal viewing distance of 10 inches (45).

Considering the second measurement first and the limitations of the human eye, it seems reasonable that a linear magnification will exist such that the operator

can perform the latter measurement to whatever degree of precision he requires provided he knows precisely the location of the most dense region of the moire fringe.

Several properties of the human eye make the selection of the most dense region of the moire fringe a difficult measurement. Intensity of light is the time average value of energy crossing a unit area per unit time. Photometric brightness or luminance is the intensity/unit area in a given direction. Visual brightness is the apparent luminance as registered by the human eye. It is a function of age, immediate environment, and general health of an observer as well as color of the light. The receptors that give the great sensitivity to the eye are located in a small central group around the optical axis. This implies that the eye can make measurements of relative intensities only within a pencil of rays 4-5 mm. or 0.2 inches in diameter (45). Michelson's criteria for the measure of distinctness of local intensities at a point are defined as visibility:

$$V = \frac{I_{MAX} - I_{MIN}}{I_{MAX} + I_{MIN}} \quad (2.2.11)$$

This is a mathematical measure to define quantitatively the visibility of local changes in intensities. It is known that a limit of the ability of the eye to separate

local brightness exists. The equivalent mathematical criteria are the Rayleigh criteria. These criteria attempt to place a limit on the spatial separation of two relative intensities of monochromatic light which can just be separated. It is known that the Rayleigh criteria exceed the limits of the eye in brightness variation detection (45).

Magnitudes of these limit criteria are not important to this discussion; indeed, they will be variable from individual to individual.

It is reasonable that there exists a limiting visibility of brightness variation that can exist in the neighborhood of a point that can be detected by the eye. Call this limit λ . Then

$$V = \frac{I_{MAX} - I_{MIN}}{I_{MAX} + I_{MIN}}, \quad V > \lambda; \quad V = 0, \quad V < \lambda \quad (2.2.12)$$

The eye scans a pencil 0.2 inches in diameter. Let this fixed field of view be $L - A$.

$$L \geq |x_1 - x_0| \geq A \quad (2.2.13)$$

Make the assumption of a sinusoidal variation of intensities between fringes of the moire pattern.

$$I = I_0 + I_1 \cos 2\pi n \frac{x}{\delta} \quad (2.2.14)$$

Then the light intensity at two neighboring points can be expressed

$$\begin{aligned} \text{at } I_{x_1} &= I_0 + I_1 \cos 2\pi n \frac{x_1}{\delta} \\ \text{at } I_{x_0} &= I_0 + I_1 \cos 2\pi n \frac{x_0}{\delta} \end{aligned} \quad (2.2.15)$$

then if $I_{x_1} \gg I_{x_0}$,

$$V = \frac{I_1 (\cos 2\pi n \frac{x_1}{\delta} - \cos 2\pi n \frac{x_0}{\delta})}{2I_0 + I_1 (\cos 2\pi n \frac{x_1}{\delta} + \cos 2\pi n \frac{x_0}{\delta})} \quad (2.2.16)$$

As $x_1 - x_0 = L$

by the limitations of the eye

$$V = \frac{I_1 [2 \sin \frac{\pi}{\delta} (2x_1 + L) \sin \frac{\pi}{\delta} L]}{2I_0 + I_1 [\cos \frac{\pi}{\delta} (2x_1 + L) \cos \frac{\pi}{\delta} L]} \quad (2.2.17)$$

then

$$V = \frac{I_1 [2 \sin \frac{\pi}{\delta} (2x_2 + L) \sin \frac{\pi}{\delta} L]}{2I_0 + I_1 [\cos \frac{\pi}{\delta} (2x_1 + L) \cos \frac{\pi}{\delta} L]} \quad (2.2.17)$$

$\delta = KL$ by means of linear magnification of the negative, L being a fixed constant of the eye ($L = 0.2$ inches). This equation shows that the visibility of whole fringes is greater when

and

$$V = \frac{I_1}{I_0} \quad (2.2.18)$$

In this condition the whole sensitive field of the eye is used, i.e., $x_1 - x_0 = L$. The greatest range of brightness can be accommodated. All receptors are working. Now due to the minimum spacing of these

receptors, the smallest distance that can be accommodated is A ($A = 1/100$ inches).

Consider the visibility of part of the fringe under these conditions;

let $x_1 = x_0 + A'$ where A' is a parameter $A < A' < L$

$$V' = \frac{I_1 [2 \sin \frac{\pi}{\delta} (2x_0 + A') \sin \frac{\pi}{\delta} A']}{2I_0 + I_1 [2 \cos \frac{\pi}{\delta} (2x_0 + A') \cos \frac{\pi}{\delta} A']} \quad (2.2.19)$$

In the moire fringe photographic negative, the most dense region occurs when $x_0 = 0$

$$V' = \frac{I_1 (2 \sin^2 \frac{\pi}{\delta} A')}{2I_0 + I_1 (2 \cos^2 \frac{\pi}{\delta} A')} \quad (2.2.20)$$

For this equation, two limits of the eye exist

$$V' \geq \lambda \quad \text{or} \quad A' \geq A$$

Note that in equation (2.2.17), when $\delta = L$, does not imply that the fringes disappear; it implies only that the viewer must change his eye position. The comparison in intensities must be made between an impression stored in the mind and a new impression. A discussion of this topic is beyond the scope of this paper.

The requirement for accurate linear measurement is that $\delta \gg L$ and $L \gg A'$ are in direct opposition to the

requirements for accurate intensity measurements by the eye. The variations in the limits λ and A and the ability of mental comparisons of intensities between individuals are the subjective portions of the analysis of the data from the moire method and from the photo-elastic method. From this discussion it will be noted that no limit can be placed on the accuracy of measurements for an individual because λ and A are variable for the individual; also, a matter of intuition and experience and prior knowledge enter into each measurement. It must be noted that with the eye as a measuring device, only limited means exist for augmenting the intensity measurement. These would involve using light of a color corresponding to the maximum sensitivity of the particular eyes to view the negative and/or an elaborate system whereby a set of eyes could become dark-oriented and then suddenly view the negative for a short time. It is known that dark-oriented eyes have a greater sensitivity to intensity variations, especially in the violet wave length (Purkinje effect).

One method of attempting to relieve the problem has been used prior to this research. The relief was obtained by recording only a portion of the data in the moire pattern by the use of slightly underexposed process

films of very great contrast. These resulted in negatives of clear blank spaces and dark black fringes. While pleasing to the eye to view as a work of art, these negatives possess the same limitations.

A prior suggestion by the author (2) was that a photocell be used for which λ does not exist and for which suitable electrical magnification of I can be obtained, thus allowing simultaneous accurate measurements of linear position and maximum density in a photographic negative.

2.3 Consideration of the Moire Method as a System

It is time to consider all experimental methods as systems whereby the input is a question asked of nature and the output is the smallest units of information that can be assembled by the experimentalist to answer the question. In experimental stress analysis, these questions of nature are usually the required stresses in some prototype under some condition of loading. The useful outputs are the stress related units that can be measured; for the moire method, these are derivatives of displacement which are components of the strain tensor.

The components of the moire method system may be listed:

(1) Suitable modeling laws between prototype and model are the first component. The geometric nature of moire as opposed to photoelasticity reduces the component to simple length and force scaling, a matter of elementary consideration and preclusion of further discussion.

(2) Suitable methods for imposing boundary conditions on the model are the second component. With moire again opposed to photoelasticity, this matter requires no special consideration.

(3) Model preparation for the use of the method, including the imposition of the grid on the undeformed model, is the third component.

Components 1, 2, and 3 involve the transformation from prototype to model and may be discussed under a general topic of modeling system.

(4) A method of comparing model and reference grid to form the moire fringe pattern is another important component. This topic could be classified as the image forming system.

(5) Since it is not convenient to analyze data directly, a memory system must be considered. Photographic recording of light intensities at a point in a field seem to be ideal for the moire method. The

possibilities should be considered that other methods could exist, such as flying spot scanning of the image while recording directly on magnetic tape.

(6) Data stored in a memory must be retrieved in order to be useful. This act of retrieving data and putting the data in a useful form is often called data acquisition. The system that accomplishes this act is called the data acquisition system. Prior to this research this was the act of reading a moire photographic negative.

(7) The data reduction system should be self-explanatory. This system performs the required transformation of data to useful output. Prior to this research this was the act of constructing curves of and graphically differentiating.

(8) Finally, the analysis system interprets the useful output in terms of the original question. This system must necessarily be the stress analyst.

Each component of the moire system is unique in that it performs a transformation. Each transformation will contain a unique set of errors. Then each component of the system must be considered in the development of the moire method and in the solution of the particular thermal stress problem selected.

CHAPTER III

THE DEVELOPMENT OF THE MOIRE METHOD IN THIS RESEARCH GENERAL CONSIDERATIONS

3.1 The Modeling System

The transformation from prototype to model required no general consideration other than the following:

(a) Induction heating would be used to achieve the high temperature boundary conditions.

(b) The lines would be imprinted on the model by use of acid etching.

A detailed discussion of these considerations will be found in the next chapter.

3.2 The Image Forming System

The transformation in this system is from a displacement field on a model to a moire fringe pattern, i.e., from strain energy to light energy. The manner of accomplishing the transformation is by superimposing the illuminated model grid and a reference grid.

Three methods for accomplishing this superposition are described in Chapter II.

The first of these methods, that is, by directly placing a reference grid in mechanical contact with the model grid, has been used by the author in previous thermal stress research (2). Two limitations were obvious with this method. The first of these was that no technique of permanently reproducing lines on fused silica could be found other than engraving with a ruling machine. The second and more serious limitation was in the development of a general method for studying thermal stress. To be a general method for studying thermal stresses, a method should embrace thermal problems involving convective heat transfer. The presence of a fused silica plate in contact with the heated model imposes a restriction on the application of the moire method to thermal problems with convective boundary conditions.

Direct photography of the model and subsequent superposition of negatives to form the moire pattern was considered. The lack of a moire pattern during the application of a load is a serious defect in this method. The test method cannot be monitored continuously.

As the result of the previous considerations, it was decided that the moire pattern should be formed by the superposition of a reference grid and the image of the model.

An integral part of the image forming system is a displacement optic law which forms the new foundations of the moire method of stress analysis.

3.3 The Moire Light Intensity-Displacement Law

One of the significant findings of the research is the application of the law relating displacement as a continuous function of light intensity in the moire fringe pattern. With this law, the moire method becomes the only experimental stress analysis method that can yield actual stress related information at a point in a two-dimensional stress field.

This displacement optic law will be reported by Sciammarella (46) while the experimental confirmation is presented by Ross, Sciammarella and Sturgeon (47) at the winter session of the Society for Experimental Stress Analysis in October 1964. In the original exposé by Sciammarella (46), only the cases of moire patterns formed by the mechanical superposition of two grids is considered.

In this writing a different approach is used to demonstrate the same optic law for the case of the superposition of an image of the model grid and a reference grid.

A brief review of the theory of the formation of images of extended objects with incoherent light will follow. Consider that a point source of light illuminates an objective lens. The image is a diffraction pattern which is a characteristic of the lens. The structure of the image can be determined if the diffraction phenomenon is known. It is known that an optical instrument behaves as a filter, allowing only certain classes of details to pass (48). The discussion is limited to circular apertures and perfect lenses.

Consider Figure 1. Let L be a perfect lens giving an image A' of a point source at A . According to the Huygens-Fresnel principle, every point M of the wave surface Σ (sphere with center at A') can be considered as a source whose amplitude and phase are identical with the amplitude and phase of the vibration produced at M by A . All points on the wave Σ emit vibrations in phase except at A' . These vibrations do not arrive in the same vibrating phase at the plane π passing through the image A' . At a general point P'

on π , the vibrations emitted from M and C' (the center of Σ) have a difference in path length $\Delta = MP' - C'P'$.

The vibrating motion at P' due to a vibration at M is

$$\Omega_p = e^{i\kappa(MP' - C'P')} = e^{i\kappa\Delta}$$

$$\kappa = \frac{2\pi}{\lambda} \quad (3.3.1)$$

or the vibrating motion at P' due to this whole wave would be:

$$U = \iint_{\Sigma} e^{i\kappa\Delta} ds \quad (3.3.2)$$

Consider Figure 2, which shows a three-dimensional view of the plane π and a position of the wave front Σ . A corresponding set of coordinates z' and y' are centered on A'. M is a point whose coordinates are (x', y') and P' has the coordinates (z', y') . The angle C'A'M' is denoted α_m and is assumed to be small. C'A' = R, $x = \overline{M\pi}$. Then:

$$MP' = y' - R\beta \quad (3.3.3)$$

$$C'P' = \sqrt{y'^2 + z'^2 + R^2} \quad (3.3.4)$$

from which by expanding and considering an isoplanic region

$$\Delta \approx \int \int_{\Sigma} e^{i\kappa(\beta'y' + \alpha'z')} d\beta' d\gamma' \quad (3.3.5)$$

and

$$U(y, z) = \iint e^{ik(\beta'y' + \gamma'z')} d\beta' d\gamma' \quad (3.3.6)$$

It has been assumed that the lens is uniformly transparent and that the amplitude of Σ is constant. This is not always true. For a general case the amplitude of Σ is a function $g(\beta', \gamma')$. If the lens has only variations in absorption, $g(\beta', \gamma')$ is real. If the lens introduces only variations in phase, $g(\beta', \gamma')$ is imaginary. In general it is complex. The amplitude of the disturbance at P' then is

$$U(y', z') = \iint g(\beta', \gamma') e^{ik(\beta'y' + \gamma'z')} d\beta' d\gamma' \quad (3.3.7)$$

The function $U(y', z')$ satisfies Dirichlet conditions and the limits of integration may be extended without bounds by defining $g(\beta', \gamma')$ on Σ and $g(\beta', \gamma') = 0$ elsewhere. Equation (3.3.7) defines a Fourier transform pair, thus,

$$g(\beta', \gamma') = \frac{1}{2\pi} \iint U(y', z') e^{ik(\beta'y' + \gamma'z')} dy' dz' \quad (3.3.8)$$

From (3.3.7) and (3.3.8) it can be seen that if the structure of the image is known, the complex amplitude of the lens may be determined or vice versa.

To illustrate the general optics problems, consider a point source of light at A. This will result in a diffraction phenomenon at the lens. If it is assumed that the lens is perfect and that the wave front from A and from A' form an isoplanic region at the lens, which is implying that α_n is small, then $g(\beta', \gamma') = 1$ within the aperture and $g(\beta', \gamma') = 0$ outside. One can say

$$\begin{aligned} \angle MA'C' &= \alpha, & \angle y'A'P' &= \theta' \\ \angle MC'B' &= \theta, & A'P' &= \rho \end{aligned} \quad (3.3.9)$$

or

$$\rho' = \alpha \cos \theta \qquad y' = \rho \cos \theta'$$

$$\gamma' = \alpha \sin \theta \qquad z' = \rho \sin \theta' \quad (3.3.10)$$

or

$$\rho' y' + \gamma' z' = \alpha \rho \cos (\theta' - \theta) = \alpha \rho \cos \theta \quad (3.3.11)$$

The relative amplitude at P' is, by an interchange of limits,

$$U(y' z') = \int_0^{\alpha_n} \int_0^{2\pi} e^{i k \alpha \rho \cos \theta} \alpha \, d\alpha \, d\theta \quad (3.3.12)$$

or

$$U(y', z') = \pi \alpha_n^2 \frac{2 J_1(z)}{z} \quad (3.3.13)$$

where $Z = K \alpha_n \rho$ and J_1 is the Bessel function. The relative illumination at P' is then (with $Z = 0$ as a reference)

$$E(y', z') = \left[\frac{2 J_1(z)}{z} \right]^2 \quad (3.3.14)$$

This is the distribution of illumination in the well known Airy disc due to a point source.

To pass from the image of a point source to the image of an extended object, the nature of the illumination of the object must be known. In this case of most optical instruments, such as instruments for observation, cameras, and spectrographs, the object can be considered to emit vibrations which are incoherent, that is, there is no fixed phase relation among the vibrations. The principal effect of a good lens is to act as a spatial frequency filter. To examine this effect for an extended incoherently illuminated object, some new notations are defined by Figure 3.

These are:

(a) $E(y'z')$ represents the intensity of the diffraction pattern with center at A' , which is the image of a point source at A . It is given by (3.3.14).

(b) A point source at B gives, at B' , the same diffraction phenomenon as A at A' but shifted a distance $A'B'$. If the coordinates at B' are equal in absolute value to those of B or by assuming a magnification equal to 1.0, the illumination at B' from a point source at B will be given by the function $E(y' - y, z' - z)$.

(c) The extended incoherent object is located at A in the yz plane and the luminous flux distribution is given by a function $O(y, z)$.

(d) The luminous flux density at the image in plane $y'z'$ will be the function $I(y'z')$.

The luminous flux density $I(y'z')$ is determined by $O(y, z)$ and $E(y' - y, z' - z)$. In fact, at each point in the image, it is the sum of the individual fluxes from each point of the object.

$$I(y', z') = \iint O(y, z) E(y' - y, z' - z) dy dz$$

(3.3.15)

A diffraction phenomenon exists on the $\rho' r'$ plane due to the function $O(xy)$. This diffraction pattern is the Fourier transform of $O(xy)$, say $\omega(\rho', r')$. Now if the variable $\mu' = \rho'/\lambda$ and $\nu' = r'/\lambda$, which are spatial frequencies, are used, the function $\omega(\mu', \nu')$ can be expressed by (3.3.8).

$$\omega(\mu', \nu') = \frac{1}{2\pi} \iint O(x, y) e^{i2\pi(\mu'y' + \nu'z')} dy dz \quad (3.3.16)$$

Similarly, by considering the image of a point source, a diffraction phenomenon exists at $\rho' r'$ characterized by the function.

$$\Omega(\mu', \nu') = \iint E(y', z') e^{i2\pi(\mu'y' + \nu'z')} dy' dz' \quad (3.3.17)$$

or

$$E(y', z') = \frac{1}{2\pi} \iint \Omega(\mu', \nu') e^{-i2\pi[\mu'(y'-y) + \nu'(z'-z)]} d\mu' d\nu' \quad (3.3.18)$$

and

$$E(y'-y, z'-z) = \frac{1}{2\pi} \iint \Omega(u', v') \epsilon^{-i2\pi[u'(y'-y) + v'(z'-z)]} du' dv' \quad (3.3.19)$$

thus

$$I(y', z') = \frac{1}{2\pi} \iint O(x, y) \left[\iint \Omega(u', v') \epsilon^{-i2\pi[u'(y'-y) + v'(z'-z)]} du' dv' \right] dy' dz' \quad (3.3.20)$$

whence

$$I(y', z') = \frac{1}{2\pi} \iint \Omega(u', v') \omega(u', v') \epsilon^{-i2\pi(u'y' + v'z')} du' dv' \quad (3.3.21)$$

or

$$\Omega(u', v') \omega(u', v') = \iint I(y', z') \epsilon^{i2\pi(u'y' + v'z')} dy' dz' \quad (3.3.22)$$

Therefore the Fourier transform of the illumination in the image (R.H.S. of 3.3.22) is equal to the product of the Fourier transform of the object and the Fourier transform of the image of a point source.

The function $\Omega(u', v')$ is of extreme importance to the practical use of (3.3.22), for it is within this term that the special frequency filtering must exist. $\Omega(u', v')$ can be calculated with the assumption of a lens free of aberrations and a circular aperture. $E(y'z')$ was a function of $U(y'z')$ and since the absorptive object was assumed,

$$E(y', z') = |U(y', z')|^2 \quad (3.3.23)$$

writing

$$\Omega(u', v') = \iint U(y', z') U(y', z') e^{-i2\pi(u'y' + v'z')} dy' dz' \quad (3.3.24)$$

and using (3.3.7)

$$\Omega = \iint U(y', z') e^{i2\pi(u'y' + v'z')} \left[\iint g(\beta', \gamma') e^{-ik(\beta'y' + \gamma'z')} d\beta' d\gamma' \right] dy' dz' \quad (3.3.25)$$

reversing the order of integrations and recalling

$$K = 2\pi/\lambda$$

$$\Omega(\mu', \nu') = \iint g(\beta', \gamma') \left[\iint U(y', z') \right. \\ \left. e^{-i\kappa [(\beta' - \lambda'\mu')y' + (\gamma' - \lambda'\nu')z']} dy' dz' \right] d\beta' d\gamma' \quad (3.3.26)$$

from which

$$\Omega(\mu', \nu') = \iint g(\beta', \gamma') g(\beta - \lambda\mu, \gamma - \lambda\nu) d\beta' d\gamma' \quad (3.3.27)$$

Now for a uniformly transparent lens $g(\beta', \gamma')$ is constant over the circle area of radius α_m (Figure 4) and $g(\beta', \gamma') = 0$ outside. The function $g(\beta' - \lambda\mu', \gamma' - \lambda\nu')$ is identical with $g(\beta', \gamma')$ but shifted by a length $\lambda\sqrt{\mu'^2 + \nu'^2}$. Since the phenomena are symmetrical, the area of integration can be shown as in Figure 5. Where $\mu' = 0$, then

$$\Omega(\mu', \nu') = \Omega(\mu'^2 + \nu'^2) = \Omega(\nu') = S \quad (3.3.28)$$

or

$$\Omega(\mu', \nu') = \frac{2}{\pi} (\theta - \sin \theta \cos \theta) \quad (3.3.29)$$

and

$$\cos \theta = \frac{\lambda v'}{z \alpha m} \quad (3.3.30)$$

The function $\Omega(\mu'v')$ can be plotted with the amplitude variation of the Fourier transform of a point source image as the abscissa and $\cos \theta$ as the ordinate.

The displacement-light intensity law for the moire method may now be considered. Consider that a model exists with lines impressed upon it such that there will be amplitude variation in the luminance from the object. Contrast can be defined if L_1 is the luminance of the most intense region and L_2 is the luminance of the least, or

$$\gamma = \frac{L_1 - L_2}{L_1} \quad (3.3.31)$$

Now if $L_1 = 1.0$ and $L_2 = 0$ and the lines have an initial spacing of p with the width of a line being p/s , the variation in luminance $O(z)$ can be written

$$O(z) = 1 + \frac{4}{\pi} \left[\sin \frac{2\pi z}{p} + \frac{1}{3} \sin 3 \cdot \frac{2\pi z}{p} + \dots \right] \quad (3.3.32)$$

the Fourier transform of $O(z)$ is $\omega(v')$ from (3.3.16)

$$\omega(v') = \frac{1}{2\pi} \int O(z) e^{i2\pi v'z} dz \quad (3.3.33)$$

Let $O(z)$ by the Euler formulas be expressed

$$O(z) = 1 - \frac{2i}{\pi} \left[e^{\frac{i2\pi z}{p}} - e^{-\frac{i2\pi z}{p}} + \frac{1}{3} (e^{i3 \cdot \frac{2\pi z}{p}} - e^{-i3 \cdot \frac{2\pi z}{p}}) + \dots \right] \quad (3.3.34)$$

then

$$O(z) = 1 - \frac{2i}{\pi} \left[e^{i \frac{2\pi z}{p}} - e^{-i \frac{2\pi z}{p}} + \frac{1}{3} (e^{i 3 \cdot \frac{2\pi z}{p}} - e^{i 3 \cdot \frac{2\pi z}{p}}) + \dots \right]$$

(3.3.35)

Then by integration and use of L'Hospital's rule it is found that ω, ν , has discrete maximum at the $\nu' = 0$ $\omega(\nu') = 1.0$

$$\nu_1' = -\frac{1}{p} \quad \omega(\nu') = -\frac{2i}{\pi} ; \quad \nu_2' = \frac{1}{p} \quad \omega(\nu') = \frac{2i}{\pi} ;$$

(3.3.36)

$$\nu_3' = -\frac{3}{p} \quad \omega(\nu') = -\frac{2i}{3\pi} ; \quad \nu_4' = +\frac{3}{p} \quad \omega(\nu') = \frac{2i}{3\pi}$$

The filtering of special frequencies by the lens can now be graphically shown if $\lambda \nu'$ is chosen as the abscissa and the amplitude of the Fourier transform is chosen as the ordinate and the curve of $\Omega(\nu')$ is superimposed (Figure 6).

Here the function $\Omega(\nu')$ is

$$\Omega(\nu') = \frac{2}{\pi} (\theta - \sin \theta \cos \theta) \quad (3.3.37)$$

and

$$\cos \theta = \frac{\lambda \nu'}{2\alpha m}$$

(3.3.38)

is an angle on the complex plane of amplitude of the Fourier transform; $\Omega(\nu')$ is a relative maximum when $\cos \theta = 0$ and $\Omega \nu' = 0$ when $\cos \theta = 1$ or $\lambda \nu' = 2\alpha m$.

Thus in photographing the model the aperture is set at αm so that

$$\lambda \nu' = \frac{3\lambda}{p} = 2\alpha m$$

(3.3.39)

For the image forming device and the light source used; that is,

$$\lambda = 5000 \text{ \AA} = 0.2 \times 10^{-4} \text{ INCHES}$$

$$P = \frac{1}{300} \text{ INCHES}$$

$$\alpha_n = \frac{\text{CIRCULAR OPERTURE DIAMETER}}{\text{DISTANCE TO THE IMAGE PLANE}} = \frac{Q}{D}$$

$$D = 24 \text{ INCHES}$$

yields

$$\alpha = \frac{3\lambda D}{2P} = 0.216 \text{ INCHES} \quad (3.3.40)$$

This is the theoretical aperture diameter to exclude the third harmonics of the spatial frequency. Practically, this limit may be exceeded because of the amplitude limiting characteristics of the function

Ω and the manner by which this function approaches zero transmission.

Considering only the first harmonic passed by the aperture:

$$\omega(v') \Omega(v') = 1 \cdot \Omega(v=0) + \frac{2i}{\pi} \left[\Omega(v=-\frac{1}{P}) e^{i2\pi(\frac{1}{P}+v')} + \Omega(v=-\frac{1}{P}) e^{-i2\pi(\frac{1}{P}-v')} \right] \quad (3.3.41)$$

and

$$I(z') = \frac{1}{2\pi} \int_{-\alpha}^{\alpha} \omega(v') \Omega(v') e^{-i2\pi v' z'} dv' \quad (3.3.42)$$

Considering (3.3.41) and $\Omega(\nu' = -\frac{1}{P}) = \Omega(\nu' = +\frac{1}{P})$

$$I(z') = 1 + \frac{4}{\pi} \left[\Omega(\nu' = \frac{1}{P}) \sin \frac{2\pi z'}{P} \right] \quad (3.3.43)$$

and

$$\Omega(\nu' = \frac{1}{P}) = \frac{2}{\pi} (\theta - \sin \theta \cos \theta) \quad (3.3.44)$$

where

$$\cos \theta = \frac{\lambda}{2P\alpha_m} \quad (3.3.45)$$

with $\alpha_m = \frac{3\lambda}{2P}$ from (3.3.39)

$$\cos \theta = 0.333 \quad \sin \theta = 0.948 \quad \theta = 70.8^\circ = 1.23 \text{ RAD.}$$

and

$$\Omega(\nu = \frac{1}{P}) = \frac{2}{\pi} (0.92) \quad (3.3.46)$$

Therefore the image is

$$I(z') = 1 + \frac{8}{\pi^2} (0.92) \sin \frac{2\pi z'}{P'} \quad (3.3.47)$$

It will be noted the contrast would be reduced. In the plane of the image (3.3.47) there exists an absorption grating which is the reference grid. If the grid alters only the amplitude of the light disturbance, it is referred to as an absorption grid and its transmission function $F(x,y)$ is real.

The transmission function is defined

$$F(x, y) = \frac{I_o(x, y)}{I_i(x, y)} = \frac{U_o U_o^*}{U_i U_i^*} \quad (3.3.48)$$

where $I_o(x, y)$ is the resulting illumination and $I_i(x, y)$ is the incident illumination. $U_o(x, y)$ is the complex amplitude and $U_o^*(x, y)$ is the complex conjugate. When the reference grid is illuminated by a constant intensity $I_i(x, y) = 1.0$, the resulting illumination is

$$I_o = 1 + \frac{4}{\pi} \left[\sin \frac{2\pi z}{p} + \frac{1}{3} \sin 3 \cdot \frac{2\pi z}{p} + \dots \right] \quad (3.3.49)$$

therefore

$$F(z) = 1 + \frac{4}{\pi} \left[\sin \frac{2\pi z}{p} + \frac{1}{3} \sin 3 \cdot \frac{2\pi z}{p} + \dots \right] \quad (3.3.50)$$

When the reference grid is illuminated by (3.3.47), the resulting image luminance is

$$I_o(z) = I(z) = F(z) I(z') \quad (3.3.51)$$

or

$$\begin{aligned} I(z) &= \left(1 + \frac{8}{\pi^2} (0.92) \sin \frac{2\pi z'}{p'} \right) \left[1 + \frac{4}{\pi} \left(\sin \frac{2\pi z}{p} + \frac{1}{3} \sin 3 \cdot \frac{2\pi z}{p} + \dots \right) \right] \\ &= \left[1 + \frac{8}{\pi^2} (0.92) \sin \frac{2\pi z'}{p'} \right] + \left[1 + \frac{4}{\pi} \left(\sin \frac{2\pi z}{p} + \frac{1}{3} \sin 3 \cdot \frac{2\pi z}{p} + \dots \right) \right] \\ &\quad + \left[\frac{4}{\pi} \cdot \frac{8}{\pi^2} (0.92) \sin \frac{2\pi z'}{p'} \sin \frac{2\pi z}{p} \right] + \left[\frac{8}{\pi^2} (0.92) \sin \frac{2\pi z'}{p'} \left(\frac{4}{3\pi} \sin 3 \cdot \frac{2\pi z}{p} \right) \right] \\ &\quad + \dots \quad (3.3.52) \end{aligned}$$

In expression (3.3.52), the first and second bracketed quantities can be recognized as the original lines. The third bracketed quantity can be written

$$\frac{4}{\pi} \cdot \frac{8}{\pi^2} (0.92) = C_1 \quad (3.3.53)$$

and

$$C_1 \sin \frac{2\pi z'}{p'} \sin \frac{2\pi z}{p} = \frac{C_1}{2} \left[\cos 2\pi \left(\frac{z'}{p} - \frac{z}{p} \right) - \cos 2\pi \left(\frac{z'}{p} + \frac{z}{p} \right) \right] \quad (3.3.54)$$

The fourth and subsequent bracketed quantities can be written

$$C_2 \sin \frac{2\pi z'}{p'} \sin 3 \cdot \frac{2\pi z}{p} = \frac{C_2}{2} \left[\cos 2\pi \left(\frac{z'}{p} - \frac{3z}{p} \right) - \cos 2\pi \left(\frac{z'}{p} + \frac{3z}{p} \right) \right] \quad (3.3.55)$$

The structure of the image can then be written

$$I(z) = \left[1 + C_1 \sin \frac{2\pi z'}{p'} \right] + \left[1 + \frac{4}{\pi} \left(\sin \frac{2\pi z}{p} + \dots \right) + \left[\frac{C_1}{2} \cos 2\pi \left(\frac{z'}{p} - \frac{z}{p} \right) \right] \right. \\ \left. - \left[\frac{C_1}{2} \cos 2\pi \left(\frac{z'}{p} + \frac{z}{p} \right) \right] + \left[\frac{C_2}{2} \cos 2\pi \left(\frac{z'}{p} - \frac{3z}{p} \right) \right] - \left[\dots \right] + \dots \right] \quad (3.3.56)$$

As this image is viewed by a second optical system such as the eye, the lens of the eye performs the same optical filtering described in (3.3.35) and (3.3.47). As the angular aperture of the eye is made

smaller by withdrawing the image, a point is reached where spatial frequencies of the order of $1/p$ and $1/p'$ cannot pass through the lens.

Considering (3.3.56) as the new object and taking Fourier transforms analogous to (3.3.36), the spatial frequencies are found to be

$$\begin{aligned} \nu_1 = \pm \frac{1}{p} \quad \nu_2 = \pm \frac{3}{p} \quad \nu_3 = \pm \frac{1}{p'} \quad \nu_4 = \pm \left(\frac{1}{p} - \frac{1}{p'} \right) \\ \nu_5 = \pm \left(\frac{1}{p'} + \frac{1}{p} \right) \quad \nu_6 = \pm \left(\frac{1}{p'} - \frac{3}{p} \right) \end{aligned} \quad (3.3.57)$$

Frequencies greater than $\frac{1}{p}$ and $\frac{1}{p'}$ cannot pass through the lens and $\frac{1}{p} = \frac{1}{p'} \pm E$ where E is a small difference. The condition is shown in Figure 7, where it can easily be seen that only $\frac{1}{p}$ will pass through the lens. Then the image on the retina is

$$I(z) = 1 + \frac{C_1}{4} \cos 2\pi \left(\frac{z'}{p} - \frac{z}{p} \right) \quad (3.3.58)$$

As the aperture of the eye is increased either by linear magnification of the scale of the image or by use of a magnifying glass, the structure of the image on the retina approaches more closely the image (3.3.56)→

Equation (3.3.58) is the displacement optic law for the moire method of stress analysis. Consider the first harmonic of the function $I(x)$

$$C \cos 2\pi \left(\frac{z'}{p} - \frac{z}{p} \right) \quad (3.3.59)$$

The function has periodic maximum when

$$\frac{z'}{p} - \frac{z}{p} = n \quad n = 0, 1, 2, 3, \dots \quad (3.3.60)$$

With the system devised in this research for obtaining moire images, z'_1 and p' are the relative linear position and pitch of the image. They are related directly to the linear dimension and pitch of the grid impressed on the model designated z_1 , and p respectively. The corresponding linear position and pitch of the reference grid are z_2 and p .

$$\begin{aligned} z'_1 &= (1 + \beta) z_1 \\ p' &= (1 + \beta) p \\ \beta &< 1.0 \end{aligned} \quad (3.3.61)$$

A point in the image belongs to z'_1 , and z_2

$$z_2 = z'_1 \quad (3.3.62)$$

therefore, in general, for maxima

$$\frac{(1 + \beta) z'_1}{(1 + \beta) p} - \frac{z_2}{p} = n, \quad (3.3.63)$$

and
$$Z_1 - Z_2 = U(z) = np, \quad (3.3.64)$$

Showing the relation with the previous theory of analysis of moire fringe data.

In the case of an initial pattern

$$Z_2 = Z_1' = (1+\beta) Z_1 \quad (3.3.65)$$

and the argument of the function for maxima is

$$\frac{Z_1}{P} - \frac{(1+\beta)Z_1}{P} = n \quad (3.3.66)$$

or

$$-\beta Z_1 = np = U(z) \quad (3.3.67)$$

Thus, all of the previous theory described in Section 2.1 of Chapter II is included within the new theory. Then, in general, the displacement-intensity law can be written

$$I = I_0 + I_1 \cos \frac{2\pi U}{P} \quad (3.3.68)$$

or

$$U = \frac{P}{2\pi} \text{ARC COS} \left(\frac{I - I_0}{I_1} \right) \quad (3.3.69)$$

Sciammarella has shown that for heterogenous displacement fields the same optical law holds. The major difference is that the Fourier transforms, instead of being discrete spikes as shown in Figure 8, are instead spikes of finite width centered on the carrier frequencies $\frac{1}{\bar{p}}, \frac{3}{\bar{p}}, \frac{5}{\bar{p}}$. Here $\bar{p}$ is the average pitch which is modulated by the local displacement.

The errors involved in the transformation from strain energy to light energy are those of poor technique. These include uneven illumination of the model and spotting due to improper line printing and photographic techniques. The image will reflect the errors in technique. These errors may be formalized in the following manner: Consider the error of uneven illumination of the model; call this error a fictitious intensity I' . Using the notation of the previous section

$$I' = I_1(z_1) \quad (3.3.70)$$

Here $I'(z_1)$ is monotonically increasing function extending from boundary to boundary of the model. The effect of I, z_1 is to amplitude modulate the function $O(z_1)$. For instance, if $I'(z_1) = \frac{A_0}{2} + \sum_{n,p} \cos \frac{2n\pi z_1}{L}$ the unmodulated function $O(z_1)$ is;

$$O(z_1) = 1 + \frac{4}{\pi} \left[\sin \frac{2\pi z_1}{p_1} + \frac{1}{3} \sin 3 \cdot \frac{2\pi z_1}{p_1} + \dots \right] \quad (3.3.71)$$

The amplitude modulated $O'(z)$ is

$$O'(z) = \left[1 + \frac{A_0}{2} + \sum_{n=1}^{\infty} C_n \cos \frac{2\pi n z}{L} \right] \quad (3.3.72)$$

$$\left[1 + \frac{4}{\pi} \sum \frac{1}{n} \sin \frac{2\pi n z}{P} \right] = \left[\left(1 + \frac{4}{\pi} \sum \frac{1}{n} \sin \frac{2\pi n z}{P} \right) \left(1 + \frac{A_0}{2} \right) + \right. \\ \left. \left[\frac{4}{\pi} \sum C_n \cos \frac{2\pi n z}{L} \sin \frac{2\pi n z}{P} \right] \right] \quad (3.3.73)$$

considering one term of the double summation

$$\frac{4}{\pi} \sum C_n \cos \frac{2\pi n z}{L} \sin \frac{2\pi n z}{P} = \quad (3.3.74) \\ \frac{4}{\pi} \sum \frac{C_n}{2} \left(\sin 2\pi \left(\frac{z}{P} - \frac{n z}{L} \right) + \sin 2\pi \left(\frac{z}{P} + \frac{n z}{L} \right) \right)$$

When the Fourier transforms for this series are computed, the new spatial frequencies

$$\nu_n = \pm \left(\frac{1}{P} \pm \frac{n}{L} \right) \quad (3.3.75)$$

result. If the aperture at the lens is reduced to pass only spatial frequencies of the order of $\frac{1}{P}$, as in (3.3.45), then n is limited to large values. The coefficients C_n for the higher harmonics will be small. Thus the moire image will contain only the terms relating to the first and last bracketed quantities in (3.3.73) above; Thus, the effect of monotonic background illumination error is to amplitude modulate the harmonics of the image.

The second type of error is that of local spotting of the negative due to poor techniques. These can be characterized by

$$I_1'' = a e^{\frac{i 2 \pi z_1' K}{P}} \quad (3.3.76)$$

where

$$z_1' \leq z' \leq z_1' + \frac{K}{P} \quad (3.3.77)$$

and $I'' = 0$ elsewhere.

Delta S is the fringe spacing. The amplitude modulation of the image (3.3.56) shown;

$$I'(z_1) = \left[1 + a e^{\frac{i 2 \pi z_1 K}{P}} \right] \left\{ \left[1 + C_1 \sin \frac{2 \pi z_1}{P} \right] + \left[1 + \frac{4}{\pi} \left(\sin \frac{2 \pi z_1}{P} + \dots \right) \right] \right. \\ \left. + \left[\frac{C_1}{2} \cos \frac{2 \pi U}{P} \right] - \dots \right\} \quad (3.3.78)$$

By use of the Euler formulae, the third term can be expressed

$$\frac{C_1}{2} \cos \frac{2\pi U}{P} = \frac{C_1}{4} C^{i2\pi \frac{U}{P}} + e^{-i2\pi \frac{U}{P}} \quad (3.3.79)$$

Considering the case of initial mismatch, $U = U' = -\beta z$, and computing the Fourier transform of the modulated term, $\omega(v')$

$$\omega(v') = \frac{1}{2\pi} \int_{-\infty}^{\infty} a \cdot \frac{C_1}{4} \int e^{i2\pi \frac{x}{P} K} \cdot e^{i2\pi \frac{\beta x}{P}} \cdot e^{i2\pi v' x} dz \quad (3.3.80)$$

new frequencies will be involved, namely $v' = -(\frac{K}{P} + \frac{\beta}{P})$

This implies that some energy (intensity) of the desired signal and all of the energy of the undesirable noise is contained in the side band frequencies $v' = \pm (\frac{K}{P} + \frac{\beta}{P})$. The term R/P is selected; that is, the apparent mismatch of the grids is

$$(1 + \beta) P = R' \quad (3.3.81)$$

and is usually less than 10 per cent or 0.10. For admission of the sidebands in a two-octave band width

$K = \pm \beta$. Thus, for a particular photograph then $K = .10$ and the spot can cover no less than 30 pitches. Since positive values of β seldom exceed 3 per cent, it can be concluded that the noise due to spots on the film will have sidebands which lie in the two-octave band surrounding the signal $I(z)$.

Some general statements can now be made concerning errors in the transformation from strain energy to light energy. The effect of uneven illumination of the model is only upon the modulus of the intensity function of the image. The effect of local spots is upon both the modulus and the argument of the intensity function but is purely localized to the region of the spot.

It will be noted that this modulation error exists in the original moire pattern and represents the largest source of error in the moire method of stress analysis. No previous analysis of the moire method has included this consideration. It will also be noted that this error exists equally for the photoelastic method of stress analysis and no previous consideration has been given.

An effort is now made to evaluate and place bounds on the errors attending uneven illumination and spotting. Consider a model undergoing uniform tension; the displacements are

$$u = \epsilon z \quad (3.3.82)$$

If no difference in the pitch exists between the model and the reference grids and even illumination is assumed, the intensity in the filtered image will be

$$I(z) = I_0 + I_1 \cos \frac{2\pi E z}{p} \quad (3.3.83)$$

An eye or film viewing the image records the relative maximum of

$$\frac{dI_z}{dz} = 0 = -\frac{I_1 p}{2\pi E} \sin \frac{2\pi E z}{p} \quad (3.3.84)$$

a maxima of I_2 occurs where

$$\frac{E z}{p} = n \quad n = 0, 1, 2, 3, \dots \quad (3.3.85)$$

and minima where

$$\frac{E z}{p} = \frac{2n+1}{2} \quad (3.3.86)$$

The average intensities are found where $I_z = I_0$

$$I_1 \cos \frac{2\pi E z}{p} = 0 \quad (3.3.87)$$

or

$$\frac{E z}{p} = \frac{2n+1}{4} \quad (3.3.88)$$

Call the initial fringe spacing $S_1 = \frac{p}{E}$

Now if a fictitious displacement u' is introduced by a difference in pitch between the model and reference grids

$$U = u' + u \quad (3.3.89)$$

and

$$u' = \beta' z \quad (3.3.90)$$

The intensity of light in the image can then be written

$$I(z) = I_0(z) + I_1 z \cos 2\pi \frac{E + \beta}{\rho} z \quad (3.3.91)$$

Call this fringe spacing $\delta_2 = \frac{\rho}{E + \beta}$

If $g' \delta_2 = \delta$, then $\beta = (g' - 1)E$ and the intensity is

$$I_z = I_0 + I_1 \cos 2\pi \frac{g'E}{\rho} z \quad (3.3.92)$$

or in general $\delta = \frac{g'E}{\rho}$

$$I_z = I_0 + I_1 \cos 2\pi \frac{z}{\delta} \quad (3.3.93)$$

From the physical optics of the image formation it was seen that the aperture performs an amplitude limiting function as well as a spatial frequency filtering. Further, the only real term in the Fourier transform is the background or average intensity amplitude. Thus, it is reasonable to expect variations

in background intensities to modulate only the first term of the image function. Modulations of the amplitudes of the second term do occur due to the lack of constant thickness of lines in either the model grid or the reference grid. In a useful model these will be of local nature. The background intensity modulating function can be expressed as

$$I_0(z) = I_0 + a_1 z + a_2 z^2 + \dots \quad (3.3.94)$$

Since it is a slowly varying function with respect to one wave length δ , the linear terms are retained

$$I_0(z) = I_0 + a_1 z \quad (3.3.95)$$

The structure of the image is then expressed

$$I(z) = I_0 + a_1 z + I_1 \cos \frac{2\pi}{\delta} z \quad (3.3.96)$$

The eye and the film record relative maxima when

$$\frac{dI_2}{dz} = 0 = a_1 - \frac{I_1 \delta}{2\pi} \cdot \frac{\sin \frac{2\pi}{\delta} z}{\delta} \quad (3.3.97)$$

or

$$\frac{\sin \frac{2\pi}{\delta} z}{\delta} - \frac{a_1 \delta}{2\pi I_1} = 0 \quad (3.3.98)$$

A convenient measure of a_1 is

$$a_1 = -\frac{KI_1}{2\pi} \quad (3.3.99)$$

Let $K/\delta = \sin 2\pi\varphi$ defining φ (3.3.100)

then (3.6.98) becomes

$$2 \sin \pi \left(\frac{x}{\delta} - \varphi \right) \cos \pi \left(\frac{x}{\delta} + \varphi \right) = 0 \quad (3.3.101)$$

when $|K| \ll |\delta|$

$$\varphi = \frac{k}{2\pi\delta} \quad (3.3.102)$$

Thus maxima occur when

$$\sin \pi \left(\frac{x}{\delta} - \frac{k}{2\pi\delta} \right) = 0 \quad (3.3.103)$$

or

$$\left(\frac{x}{\delta} - \frac{k}{2\pi\delta} \right) = n \quad n = 0, 1, 2, 3, \dots \quad (3.3.104)$$

then

$$x - \frac{k}{2\pi} = n\delta = \frac{n\rho}{\beta + E} \quad (3.3.105)$$

$$z = \frac{n\rho}{\beta+E} + \frac{k}{2\pi} = z_{\text{CORRECT}} + e_{\text{ERROR}} \quad (3.3.106)$$

Minima occur when

$$\cos \pi \left(\frac{z}{S} - \frac{k}{2\pi S} \right) = 0 \quad (3.3.107)$$

that is when

$$\frac{z}{S} - \frac{k}{2\pi S} = \frac{2n+1}{2} \quad (3.3.108)$$

$$z = \frac{2n+1}{2} S + \frac{k}{2\pi} \quad (3.3.109)$$

Thus the absolute value of the error in the position of z at points of relative maxima and minima in the image is found to be

$$|\Delta z| = \frac{k}{2\pi} \quad (3.3.110)$$

Here K is a parameter which is fractional multiplier of I , in the length of one fringe; see Figure 9. The use of mismatch between grid pitch and model pitch

can now be seen. The modulating function is given by (3.3.94) and over the face of the model describes some monotonic function caused by uneven illumination. The use of mismatch effectively divides this function into intervals such that K is decreased. Thus in the use of mismatch, more information about displacements is obtained as demonstrated by (3.3.92) and obtained with greater accuracy. It will be noticed that the error is of alternating sign at points of relative maximum and minimum and vanishingly small at the average intensity points. The magnitude of the error depends only on the local change in intensity over one fringe spacing.

It may be fruitful to try to place some bounds on the error for the tension model described in this section. The displacement is given by (3.3.82) and is the value of the hypothetical uniform strain. If the moire fringe pattern is scanned, as was done previous to this research, and some initial mismatch is taken into consideration with and without uneven illumination a measure of the error in $\mathcal{E}$ can be attained. The error in $\mathcal{E}$, by measuring between two successive maxims, say $n = 0$ to $n = 1$, can be shown by considering (3.3.106)

$$\Delta \mathcal{E} = \mathcal{E}' \text{ (modulated)} - \mathcal{E} \text{ (unmodulated)} \quad (3.3.111)$$

In general

$$\Delta E = \frac{n\rho}{z-k} - \beta - \frac{n\rho}{z} + \beta \quad (3.3.112)$$

$$\Delta E = n\rho \left[\frac{2\pi}{2\pi z - k} - \frac{1}{z} \right] \quad (3.3.113)$$

$$\Delta E = n\rho \left[\frac{k}{z(2\pi z - k)} \right] \quad (3.3.114)$$

and if

$$\beta = (g' - 1)E \quad (3.3.115)$$

$$\frac{\Delta E}{E} = g' \left[\frac{k}{2\pi z - k} \right] \quad (3.3.116)$$

This relative error can be very great. Thus measurement between successive maxima points only should be avoided. Plotting of successive points improves the accuracy for (3.3.116) and inclusion of minima points introduces error of opposite signs so that the subsequent averaging curves giving equal weight to points greatly reduces the errors due to uneven illumination, provided K is small and does not vary greatly between successive fringes. The factor K is a variable inverse function of g' , and it is reasonable to expect some optimum g^* exists for each case of uneven

illumination. These errors are also greatly reduced by analysis by the difference method provided the illumination of the model is not changed during the time interval in which the data is taken. These errors are maximum at the points of relative maximum and minimum and become vanishingly small at points near the background illumination. No use of these points was made in the previous method of analysis of moire images.

Amplitude modulation of the second term results in errors of precisely the same type as background modulation with the addition of another term representing a progressive amplitude increase with succeeding maxima. The errors are again characterized by alternating signs and vanishingly small errors at points near the background intensity. The progressive term in the expression for dI/dz is of the form

$$az \cos \frac{2\pi z}{S} \quad (3.3.117)$$

and the remaining terms are of the type shown (3.3.101). Thus the effect of the progressive term is to reduce the phase modulation of the intensity or to reduce the error.

Bounds cannot be established in general for these errors since they are functions of particular illuminations and line printing techniques. The means

to remove the effect of these errors is apparent. The principal means is to use the displacement-intensity law in the analysis of moire data whereby use can be made of the multitude of points possessing very small errors. The second means is to take advantage of the alternating sign of the errors by local averaging in the analysis process. The use of mismatch, or a difference in pitch in model and reference grid and analysis by the difference method is mandatory for all cases where uneven illumination is expected.

3.4 The Memory System

The only memory system that has received attention for use with the moire method of stress analysis has been direct photography of the moire pattern. The elements of a camera are sufficiently described (49) as to warrant little discussion other than the following.

(1) The lens used either for image formation or the photography of the fringes should be of high quality; that is, free from aberration, relatively long focallength, and the maximum amount of color correction. The latter term implies that most wave lengths of light will focus at the same point and this requires a multi-component lens such as an apochromatic lens.

(2) The aperture setting for a moire exposure is governed by one consideration not common to normal photography. This consideration is the control of the filtering action of the aperture on the spatial frequencies of the image. As shown by the calculations in Section 3.3, the setting is a compromise to admit the maximum of the first harmonics of the spatial frequencies of the object. The usual consideration of the use of aperture for the control of depth of field is accomplished by the initial focus required by the moire method. Exposure times usually controlled by the aperture setting, are controlled by adequate lighting and film selection.

(3) The relative motion between the model and the camera should be adjustable in all six degrees of freedom to allow the removal of the errors discussed in Section 2. The camera itself should be a rigid, highly damped mechanical device allowing precise individual adjustment of the plane of the lens and the plane of the image. These adjustments are required to be only along the optical axis of the instrument, with the exception that the image plane be rotatable about the optical axis. The camera should be carefully constructed to insure a complete parallelism between the lens and the image plane. The plane of image formation and the plane of the film must coincide.

If a separate camera is used for image recording in the manner of Theocaris (44) these same general requirements must be met by two systems with the additional requirement of adjustments of about five degrees of freedom between the image forming plane and the recording camera. It must be noted from the theory of image formation that use of the second camera will result in images of reduced contrast.

It must be emphasized that in any system to record separately the complete moire pattern formed, either directly by superposition or indirectly by imaging, an error in perspective may enter due to the lack of parallelism of the recording film plane with the plane of the moire pattern. This error is of the nature described in Section 2 and results in a distortion of the displacement field. The error is considerably more serious because no mechanism exists to detect it other than coarse measurements made by the eye.

Use of the imaging and recording system described in this research plus analysis by using the difference in moire patterns effectively eliminate this error. The use of the difference in moire patterns has been previously described by Dantu (38), who referred to it as the "differential method." Sciammarella (40) preferred the term "mismatch method," and recently

the method was reannounced as a new discovery by Theocaris (44) as the "differential method." The term "difference method" will be used in this work with the broader implications that the initial pattern may be composed of fictitious displacement due to many causes.

The last element of the memory system, as used, is consideration of the photographic process and selection of films to be used to affect the transformation from image to memory, i.e., light energy to chemical energy. In the application of the moire method, it may be useful to consider illuminating the model with light of various wave lengths. Principally among these will be ultraviolet with wave lengths from 136 to 3,900 Angstrom units; visible light, 3,900 to 7,600 Angstrom units; and infrared light, 7,600 to 5,000,000 Angstrom units. Photographic films are available to record intensities in each of these wave length bands. Ordinary films can be used for the ultraviolet and visible light range by virtue of the spectral sensitivity of silver bromide, the main light sensitive constituent of these films. Infrared films consist of silver bromide with a sensitizing dye which extends the maximum upper limit wave length of infrared film to about 13,500 Angstrom units, infrared light (50).

The explanation of the action of light on silver bromide has its foundations in quantum mechanics (49). It is sufficient to recognize that the action is initiated in very small (10^{-6} cm.) clumps at the corners of the silver halide crystals. The number of activated clumps depends on the intensity of light. The photographic film is composed of a very fine layer of silver halide crystals embedded in a gelatin upon a stable base of plastic or glass. The mechanism of image retention is thus:

(1) Light passing through the photographic negative will cause activated clumps of ions at each point directly proportional to the intensity and time of exposure.

(2) In the subsequent chemical development, the gelatin swells and becomes porous thus allowing passage of the developing solution to the halide crystal. The property of the developing solution is that it imparts the specific energy necessary for the metallic silver to be reduced at each clump.

(3) The fixing solution dissolves all unactivated silver halide crystals, allowing easy removal of all but the metallic silver from the emulsion during the subsequent wash.

(4) Upon drying, the gelatin shrinks back to its original shape freezing the position of each metallic silver grain to its original position with respect to the stable base of the film.

Since the mechanism is non-linear the behavior of each type of film is described by characteristic curves. It must be said that within each type of film used in this work, a remarkable uniformity in response of individual film was observed. The characteristic curves are used to relate a variable called photographic density D with a variable denoted as exposure E . Thus, exposure E is defined

$$E = It \quad (3.4.1)$$

Where I is the intensity of the light at a point and t is the total exposure time. In the normal use of film in the moire method, there is reciprocity in this relationship. For other uses, the reciprocity may not exist, depending upon the film. The term density, D , is related directly to the amount of silver deposited at a point in a photographic negative. A typical characteristic curve is shown in Figure 10. Here density, D , is plotted against the logarithms of exposure E . There are four sections of this curve that are of interest. The first is from C to A , in which no

density results from exposure. This represents an inertia of the film and is a principal factor in determining exposure time for the films. Next, there is the non-linear portion called the toe, A-B, extending to a maximum density of 0.1 for all of the films normally used in moire work. The portions of the characteristic curves useful to moire work is the linear portion B-C. At exposures greater than E_C , the density relation is again non-linear. In the linear region of a characteristic curve

$$D = \gamma \log_{10} E \quad (3.4.2)$$

Gamma, γ , is the slope of the curve in the region B-C and is a variable of development for most films. Specific control of γ for a particular film is a matter of developer used, to a lesser extent the temperature of the developer, and time of developing. If development to a specific gamma is required, films that have relatively flat gamma time curves for a specific developer in the region of interest may be selected from information supplied by manufacturers. Exact control of developing time will yield consistent results. Films such as high resolution plate or Kodalith possess very high gamma, large inertia, and little toe in their characteristic curves.

Moire patterns generally have little contrast, that is, γ' is low. Photographing moire patterns with films of this type augments the contrast but generally loses the displacement information to the inertia of the film. Specific requirements for the selection of films will be established in the next section. Characteristic curves and gamma-time curves for some films that have been used in the development of the moire method are shown in Figure 11.

For a negative correctly exposed and developed to a specific gamma, the recorded information is in the form of silver deposits which are measured by the photographic density D . Thus, except at local spots,

$$D = \gamma \log \left\{ [A(z_i') + B z_i' \sin \frac{2\pi z_i'}{\rho}] + [C(z_i) + D(z_i) (\sin \frac{2\pi z_i}{\rho}) + \dots] + [E(z_i') + F(z_i) \cos \frac{2\pi z_i}{\rho}] + \dots \right\} \quad (3.4.3)$$

Thus far, in the development of the moire method to measure thermally induced strains, the system of obtaining the image and recording the image have been examined. The optimum method of formation of the image has been developed and the corresponding displacement optic law has been discussed. The major

contribution of this research is the development of the method of analyzing the data in a completely objective fashion. This information is found in the following two sections.

3.5 The Data Acquisition System

In this research a mechanical optical instrument was designed and built. The machine is a photograph reading machine which performs two useful functions in the acquisition of data from a moire photograph. First, it performs the transformation from $F(z)$ to $g(t)$ in an exactly linear fashion. The purpose for this transformation is to make available a host of precision measuring devices that exist in the time domain. The second transformation is from the stored silver pattern of the photographic negative to an electric signal directly proportional to the intensity of light which caused the negative. The first transformation is characterized

$$Z = K t \quad (3.5.1)$$

The manner in which the second transformation takes place can be explained as follows: The light falling on a photocell E_c is directly proportional

to the transmission T_N of the film negative.

$$E_c = k T_N \quad (3.5.2)$$

thus

$$\log_{10} E_c = \log_{10} T_N + \text{CONSTANT} \quad (3.5.3)$$

opacity, O_N , is defined

$$O_N = \frac{1}{T_N} \quad (3.5.4)$$

Photographic density D has another definition by virtue of Lambert's law of absorption

$$D = \log_{10} O_N = -\log_{10} T_N \quad (3.5.5)$$

From (3.5.5) and (3.4.2)

$$-\log_{10} T_N = \gamma \log_{10} I + \text{CONSTANT} \quad (3.5.6)$$

The resistance of the photoconductive cell is related to the intensity of incident light by

$$R = k E^{-\xi} \quad (3.5.7)$$

or

$$\log_{10} R = -\xi \log_{10} E + \text{CONSTANT} \quad (3.5.8)$$

If (3.5.3) is combined with (3.5.6) and (3.5.8)

$$\log_{10} R = \xi \gamma / \log_{10} I + \text{constant} \quad (3.5.9)$$

thus

$$R = KI^\gamma \quad (3.5.10)$$

Now this specific photocell ξ can be measured accurately for the CdS_c cell chosen $\xi = 0.95$. Gamma is a variable of the film development process as discussed in Chapter III, Section 4, and gamma is chosen so that

$$\xi \gamma = 1.0 \quad (3.5.11)$$

then

$$R = KI \quad (3.5.12)$$

The resistance of the photoconductive cell is directly proportional to the intensity of light causing the negative or the intensity of the image itself. The intensity of the image may contain information concerning the moire fringes only or moire fringes and the lines.

To be useful the transformation must also conform to requirements of the displacement-optic law. The photocell must be exposed to the image through an optic system of small angular aperture to accomplish the necessary filtering of spatial frequencies. The result would then be.

$$I = I_0(x) + I_1 \cos \frac{2\pi v}{\rho} \quad (3.5.13)$$

due to the noise of uneven illumination.

The transformation of intensities to the resistance of the photocell, according to the displacement-intensity law, can be accomplished in several ways. While the initial imaging of a model and the formation of the moire pattern is a problem of incoherent illumination, the transformation of the photo-reader is a problem of coherent illumination. With incoherent illumination, the light from various points of the body is assumed to be varying greatly in phase, and imaging is accomplished by scalar addition of intensities as shown in a previous section. With coherent illumination, the imaging is essentially of vector interference of the light from a single source.

The simplest method to perform transformation is use a single circular aperture in close proximity to the film which is illuminated by parallel coherent light. At each plane behind the opening there will exist a resultant diffraction pattern determined by the vector addition of light from all points of the film admitted by the opening. The photocell responds to intensities which are energy densities. The resulting resistance of the photocell is the integral of resultant intensities in the defraction patterns. If the intensi-

ties in the original image are given by (3.3.56), the resistance of the photocell is given by

$$R(z) = KI(z) = K \int_S I_s ds \quad (3.5.14)$$

$S = \text{APERTURE}$

If the limits of integration are set by restricting the diameter of the opening to np , where $n = 1, 2$ or 3 , then these contributions to the image from the lines can be vanishingly small. The integral of the slowly varying functions v and I_0 can be replaced by their central values and the resistance of the photocell is then

$$R(z) = K \left[I_0 z + I_1 \cos \frac{2\pi v}{p} \right] \quad (3.5.15)$$

This solution works very well for instances when the recording of the moire image has been separate from the formation of the image and proper apertures have been used. The apertures required involve long exposure times with the most rapid films available.

A more general solution is found in Abbe's theory of imaging of coherent extended objects. Abbe's theory of imaging requires a lens to be placed near the negative,

which is illuminated from behind by parallel coherent light. Each of the sinusoidal of the image (3.4.6) will yield successive images of the point sources on the focal plane $\gamma\beta'$ of the lens. These images are called the orders. The only requirement to eliminate any particular sinusoidal component in the image formed by a second lens placed at $\gamma\beta$ is that all higher orders than the first be not admitted. This is accomplished by placing a circular aperture on the second lens such that the angular aperture a_n is

$$a_m < \frac{\lambda}{p} \quad (3.5.16)$$

For the photo-reader, a lens was selected such that the principal focus occurred at $D = 0.500$ inches. The shortest wave length of tungsten light at the operating temperature is approximately 0.2×10^{-4} inches; thus, to eliminate the lines from the image, a circular opening is required

$$a_{m=D} = \frac{\lambda}{p} = \frac{0.2 \times 10^{-4} \times 300}{1} \quad (3.5.17)$$

or

$$a = 0.003'' \quad (3.5.18)$$

To eliminate the third harmonic of the displacement function a circular aperture (with an initial fringe spacing of 0.30 inches)

$$a = \frac{0.2 \times 10^{-4} \times 2}{0.10} = 4 \times 10^{-4} \text{ INCHES} \quad (3.5.19)$$

The latter consideration is included to allow the photo-reader to accommodate films for which the proper aperture has not been used. While apertures of the order of size of (3.5.18) can be machined, there are practical difficulties involved in creating apertures of the size of (3.5.19). Apertures of this size do exist as porosities in medium fine paper. A convenient aperture then consists of a small diameter hole of 0.003 inches covered with a porous paper. The resulting resistance of the photocell, considering the central values, then is

$$R(z) = k(I_0(z) + I_1(z) \cos \frac{2\pi u}{\rho}) \quad (3.5.20)$$

It is interesting to consider the possibility of detecting only the background illumination term $I_0(z)$ since it has been shown this may be modulated by uneven illumination. Two possibilities exist. The first of these is to establish a fixed mismatch or initial

fringe spacing upon which the deformation will appear as frequency modulations, thus establishing a field of view for a second photocell which will cover n fringes. Integration by the photocell will then yield the background intensity. The second method is to find a very dense, porous media which will have apertures of the order of 10^{-5} inches. This, used in a coherent imaging system, would block the first harmonic of displacement information if the initial fringe spacing is controlled.

As the photocell scans the negative, the two transformations occur simultaneously. If the variable resistance given by (3.5.15) or (3.5.20) is made the active arm on a Wheatstone bridge circuit, the open circuit voltage output will be

$$e = e_o(t) + e_i(t) \cos \frac{2\pi u}{p} - e_K \quad (3.5.21)$$

where e_K is an arbitrary constant offset voltage introduced through the bridge. Thus, the transformation from the chemical energy of the stored image to electrical energy of the output signal is effected.

One source of error in this transformation is that due to the product $\gamma \xi \neq 10$. Considering

$$R = R(\gamma\xi) = R\gamma$$

$$\frac{dR}{R} = \xi \log_e I d\gamma \quad (3.5.22)$$

Now

$$\frac{\log_e I}{\log_e 10} = \log_{10} I \quad (3.5.23)$$

and

$$D = \gamma \log_{10} I \quad (3.5.24)$$

So

$$\frac{dR}{R} = \frac{\xi D}{\gamma \log_{10} I} \frac{d\gamma}{\gamma} \quad (3.5.25)$$

For moire photographs $D_{MIN} = 0.1$ and $D_{MAX} = 0.3$, found in actual measurements, Therefore, equation (3.5.21) shows clearly the relative insensitivity of the device for errors in development.

The effect of the photocell development error is as follows

$$\frac{dR}{R} = \frac{0.95 \times 0.3}{2.302} \cdot \frac{d\gamma}{\gamma} = 0.124 \frac{d\gamma}{\gamma} \quad (3.5.26)$$

It is desired to use the displacement-intensity law to obtain a continuous function $\mathcal{V}$. The magnitudes of the errors due to the photocell and improper film development at various points of the curve have been shown. It will be shown now how the effect of these errors is eliminated.

When the background variation has been subtracted from the recorded signal and in the absence of photocell film error, the remaining curve would be, through the choice of a convenient origin,

$$y = A \sin \theta \quad \theta = \frac{2\pi \mathcal{V}}{\rho} \quad (3.5.27)$$

It is convenient to normalize this curve to solve for θ .

$$\bar{y} = \frac{y}{A} \sin \theta \quad (3.5.28)$$

When a film development error is present, the recorded signal is considered to be

$$y' = y + dy = A' \sin \theta' \quad (3.5.29)$$

$$A' = A + dy \big|_{\theta = \frac{\pi}{2}}$$

Again normalizing

$$\bar{y} = \frac{y'}{A'} = \sin \theta' \quad (3.5.30)$$

When

$$\bar{y} = 1.0 = \frac{y}{A} = \frac{y'}{A'} \quad \text{and} \quad \theta = \theta' \quad (3.5.31)$$

When

$$\bar{y} = 0 = \frac{0}{A} = \frac{dy|_{\theta=0}}{A+dy|_{\theta=\frac{\pi}{2}}} \quad \theta \neq \theta' \quad (3.5.32)$$

Thus the error in θ is periodic with maximum at $\theta = n\pi$ and zero at $(\frac{2n+1}{2})\pi$. The greater accuracy of points near the background curve has been shifted by the normalization to points of maxima and minima.

The error (3.5.32) is caused by the photocell-film combination, which can be evaluated from (3.5.22)

$$\frac{dy}{y} = k \frac{dR}{R} = \beta dr \log_e I \quad (3.5.33)$$

$$y = CI = A \sin \theta \quad (3.5.34)$$

k and C are constants. Thus, considering dy is constant for any particular film

$$\frac{dy}{y} = \frac{\beta dr}{k} \log_e A \sin \theta \quad (3.5.35)$$

and

$$\frac{1}{2\pi} \int_0^{2\pi} \frac{dy}{y} d\theta = \frac{\beta dy}{k 2\pi} \int_0^{2\pi} \log_e A \sin \theta d\theta = 0 \quad (3.5.36)$$

From the analysis it will be seen that local averaging over the length of curve equal to δ removes the effect of the photocell error.

3.6 The Data Reduction System

Standard methods of computer data reduction, both analog and digital, were considered to reduce the moire data in an objective fashion. Two analog computers were available on a trial basis for short time intervals only. One of these was equipped for voltage comparison and high speed repetition. An IBM 709 and a RW 300 digital computer were available. The RW 300 is a slow speed, limited memory machine used for process control; however, it does possess an analog to digital conversion section. The object of this portion of the research was to prove the feasibility of a completely objective reduction of moire data.

The optimum solution to the problem of objectivity would be to have a self-contained data reduction system

operating in real time so that, as the photograph is scanned, the components of strain would be plotted in an analog form as the useful output. The optimum system would be composed of two essential subsystems; one, a data correction system and, two, an analysis system. The analysis system would simply solve the equation

$$v = k \arccos \bar{e}(t) \quad (3.6.1)$$

where $\bar{e}(t)$ is the corrected analog voltage from the photo-reader. The analog voltage, being given by

$$e(t) = e_o(t) + e_s(t) \cos \frac{2\pi u}{p} + \varphi(t) - e_k \quad (3.6.2)$$

the required data correction to effectively remove the

$e_o(t)$ and normalize the second term is $e_s(t) \rightarrow 1.0$

The corrected analog voltage was reconstructed to preserve especially the integrity of the low error points near the background voltage $e_o(t)$. Once the function u is determined, the components of strain can be determined by differentiation by finite differences.

The order of accuracy possible in the overall moire method is an error within 1 per cent under optimum conditions. This led to the conclusion that the computer system chosen must be capable of accuracies of the order of 0.1 per cent. This is the practical lower limit of

accuracy for an analog computer. The information is "narrow band width," that is, of a long period and is thus naturally suited to digital computation.

The optimum digital program to reduce the data in the moire method would start with an electric signal (3.6.2) from the photo-reader. The following steps would then be performed:

(1) The voltage would be linearly amplified to achieve the maximum sensitivity of the analog to digital conversion which follows.

(2) The signal would then be introduced into a digital data acquisition system such as the Dymec 2013R. This system digitizes the analog signal and prints the results on punched tape. Sampling rates of three readings per second would be satisfactory from the viewpoint of accuracy and cost.

(3) The punched tape information would be converted to punched cards for IBM 709 input by an IBM 1401 tape-to-card converter.

(4) The computer program and the converted data would be processed in the IBM 709.

(5) The output information would be obtained in analog and digital form from the IBM 1401 computer, which is used as the print out mechanism for the 709.

The computer program for the 709 would perform the following functional operations as shown in Figure 9.

(1) Successive maxima and minima are found, denoted X_i on Figure 9.

(2) From step 1 the points A_0 of the background intensity curve are located. A_0 is the midpoint between successive X_i .

(3) For each one-half cycle, a chordal approximation to the background intensity function is used to subtract the background variations from the recorded signal.

(4) For each one-half cycle, the new maxima and minima are found. Each recorded value of the corrected data is divided by the maximum value for the one-half cycle to which the recorded value belongs. Thus a normalized curve results.

(5) The solution to

$$v = \text{ARC COS } \bar{e} \quad (3.6.3)$$

can be found by several standard routines.

(6) The curve of v is averaged to remove remaining image errors by the technique for locating a trend in data; that is by computing a local average at each point.

$$\bar{u}_k = \frac{1}{S} \sum_{i=k-\frac{S}{2}}^{i=k+\frac{S}{2}} u_i \quad (3.6.4)$$

(7) The derivative is determined by computing the central finite differences

$$\frac{d\bar{u}}{dz} = \frac{1}{2h} [\bar{u}(r+h) - \bar{u}(r-h)] \quad (3.6.5)$$

from the averaged information of $\bar{u}$.

(8) The derivative information will contain local irregularities due to spots as mentioned previously; therefore, it is smoothed by the same procedure used for this information of .

(9) The curves of data, corrected data, $\bar{u}$, corrected $\bar{u}$, derivatives, and corrected derivatives are plotted by the computer.

CHAPTER IV

THE DEVELOPMENT OF THE MOIRE METHOD IN THIS RESEARCH SPECIFIC CONSIDERATIONS

4.1 Development of the Modeling System

The thermal aspect of the problem of the research required no modeling law considerations. The models were 5.50 inches O.D. and with I.D. varying from 0.50 to 2.50 in steps of 0.50 inches. The material used was stainless steel 304, chosen for the known temperature variations of the physical constants E , ν , and α of the material, Figure 12.

The thermal boundary conditions for this problem were in axis symmetric radial temperature variation. Since the goal of the research was to attain temperatures of 1500°F with high temperature gradients, induction heating was chosen to impose the thermal load. An induction coil composed of square 1/4 inch copper tubing was wound to be just large enough for the insertion of a radiant heat insulator between the model O.D. and the coil I.D. The frequency of the induction heating oscillator was chosen to heat the model to a depth of 0.030 inches uniformly around the outer circumference. A center water cooled hub supplied the sink for the flow of heat through the model.

The radio frequency energy was controlled by varying the plate voltage of the oscillator. The temperature field imposed by induction heating was axisymmetric. The maximum circumferential variation found during the test was $< 1^{\circ}\text{F}$. The thermal boundary conditions were varied by changing the power flow in the induction coil in increments. At each change the model was allowed to attain complete equilibrium before data was taken. Convection was corrected by the use of a complete enclosure including a fused silica window through which the model could be viewed. The effects of radiant heat transfer from the model were thought to be insignificant. In this problem the errors caused by the transformation from prototype to model conditions can only be the lack of accurate determination of temperature. Temperatures were recorded by the use of a Leeds and Northrup potentiometer. Thermocouples were calibrated in place. The error involved in this phase was less than 1 per cent.

The models were prepared by normal machining techniques. The faces upon which the grids were to be imposed were finished with a medium fine surface grinding in a concentric pattern. Care was taken at the time to use slow machine feed and adequate cooling to avoid premature stress.

In order to accumulate in one place a record of the various techniques available to print temporarily or permanently the lines upon the model the following ways are recorded:

(1) Dantu (37) has described the technique of spraying a stripping film copy of the grid with white acrylic paint, then cementing the stripping film to the model, paint side down. Subsequent removal of the film backing and dissolving of the thin base with acetone leaves the emulsion exposed upon the model against a white background. This method is not suitable for thermal stress.

(2) Dantu (38) also describes the technique of printing lines of the grid with photoengravers' materials and etching an aluminum surface with a sodium hydroxide solution.

(3) Sciammarella (40) has described the method of obtaining good line master grids and a procedure for casting epoxy and urethane on the face of stripping film. The film backing is removed to obtain lines in a sheet of plastic which may be cut into models.

(4) Sciammarella and Ross (2) have described the technique of using Kodak Photo Resist K.P.R. directly to print lines on metals useful to 600°F. A technique of depositing copper between the K.P.R. lines and subsequent removal of the K.P.R. is also given.

(5) Theocaris (44) describes the use of "light" nitric acid solution to etch low carbon steel between the lines of a photoengraver's lacquer.

(6) During the course of this investigation it was found that Pitman's "Black line process" could be used to print photographically lines on plexiglass.

(7) The model grid can be impressed upon stainless steel by chemical etching. This process will be described in detail because it was used for this series of tests.

The models were washed carefully using soap, water and alcohol. When thoroughly dry, K.P.R. was sprayed uniformly over the surface and dried. A crossed master grid of 300 lines per inch on a photographic plate was held by vacuum in contact with the surface of the model during an exposure to a 20 watt arc lamp at 2 feet distance for 4 minutes. After development of this K.P.R., the models were electrolytically deep-etched in a solution of 33 per cent phosphoric acid, 57 per cent glycerin and 10 per cent water for a time of approximately three hours each. The model was made anodic and the stainless steel tank was cathodic for an impressed current density of 72 amp/sq. ft. The lines of the model grid were then checked against a grid master to assure that no error entered during the phase of the modeling system.

4.2 The Image Forming System

The model grid was superimposed on the reference grid on an intermediate plane determined by a lens system. This was achieved by altering an existing precision enlarger, Figure 13. The alterations consisted mainly of the addition of a precision apochromatic lens of long focal length and removal of the apparatus in the negative plane of the projector. A new back was fitted to the unit to convert it to a camera. Suitable mechanisms existed to adjust the back and front of the camera very precisely. The image forming screen and film holder can be rotated 360° about the optic axis of the instrument. The light from the model passes through a reference grid at the image forming plane. A removable, very finely ground glass serves to form the image of the moire pattern for visual inspection. A film negative replaces the ground glass to record the image.

A consideration of the remainder² of this moire system that will be discussed later resulted in the decision to use an initial mismatch or difference in grid and model pitches. This mismatch could be accomplished very accurately by first adjusting the model image to full size upon the image screen. The adjustment is observed by the vanishing of the fringes of the moire

pattern. A simple adjustment of the front of the camera then allowed mismatch of sufficient amount within the depth of field of the objective lens. This adjustments are greatly facilitated by consideration of the results of (2.1.14) and (2.1.15). Coincidence between the axis of symmetry of the model and the optic axis of this model is assured by the consideration of (2.2.5). Removal of curvatures from the initial pattern implies precise coincidence of the axis.

Focusing a precision camera involves the simultaneous solution of :

$$M = \frac{D_2}{D_1} \quad (4.2.1)$$

$$\frac{1}{D_1} + \frac{1}{D_2} = \frac{1}{F} \quad (4.2.2)$$

Where M is the linear magnification, D_1 is the distance from model to lens, D_2 is the distance from lens to image plane and F is the focal length of the lens. Given M and F, D_1 and D_2 can be found. Because the optical center of the lens is not known exactly, D_1 and D_2 must be carefully adjusted to yield the exact focus.

Within the previous discussion of a specific image forming system are the specifications for a general system. Techniques of adjustment and focus are included. It is understood the errors described in (2.2.6) and (2.2.7) can exist in the image forming system.

4.3 The Memory System

The problem of photographing a moire model undergoing thermal stress at high temperatures had not been solved prior to this research. Dantu(38) and Theocaris (44) reported their inability to achieve a moire fringe pattern when the model was glowing, that is, at temperatures above 900°F.

The thermal problem stems from Planck's radiation law and Wien's displacement law. The consequence of Planck's law is the well known fact that bodies emit radiation independent of material but dependent only on the absolute temperature. Wien's law describes the general shift of maximum wave length of the radiation of a black body with absolute temperature. From Figure 14 it can be seen that the practical thermal stress problem would involve radiations somewhere between 1,300°K and 2,850°K. At these temperatures the radiation is limited to wave lengths longer than 0.4×10^{-4} cm. This corresponds to the upper limit of the ultra violet range. Obviously, one solution is to restrict the illuminating source to wave lengths greater than 0.346×10^{-4} cm. and less than 0.40×10^{-4} cm. The lower limit was determined by the harmful effects of ultraviolet light on skin when wave lengths are shorter than 0.346×10^{-4} cm. light

sources are available to accomplish this. The upper limit was affected by use of a Kodak ultraviolet filter which eliminated all wave lengths above 0.4×10^{-4} cm. There is one restriction to the use of this system and that is the camera lens must be made of glass which will not absorb this wave length. Such lens exist but were not available for this research.

The solution found for this research which appears to be suitable for use to about 2500°F consists of the use of a cold light source with predominant radiation wave length of 0.5×10^{-4} cm. Such a source is available from photographic suppliers. The intense illumination of the model with this wave length masked the black body radiation completely to a temperature of 1500°F . Consideration of the film for use in thermal stress problems removed the last barrier.

In Figure 15, the spectral response of various films are shown. The fundamental response of silver bromide emulsions is shown on the non-chromatic film. Commercial orthochromatic films have a sharp cut off in response at 0.58×10^{-4} cm. and are completely insensitive to infrared radiation. Therefore, the use of orthochromatic film and the actinic light source mentioned allowed photography of a brightly glowing model at 1700°F .

The efficiency of the actinic light source alone can be estimated by the fact that panchromatic films with a limited response in the infrared region could be used to temperature above 1000°F.

A third solution to the problem was contemplated; it involved using the difference in emissivity factor for radiating black bodies. For instance, the emissivity of carbon is approximately 1.0 and stainless steel is about 0.78. Thus, if the engraved grooves on the model were filled with carbon a contrast is amplitude of the black body radiation should be visible although the temperature of the surface is uniform. Infrared film would be used with a filter to remove all wave lengths shorter than 0.7×10^{-4} cm. Preliminary tests indicated the feasibility of the method although no development was attempted. This should be a method applicable to very high temperatures.

4.4 The Data Acquisition System

The data in the moire pattern is transformed by the photo-reading device to a useful electric signal. The photo-reading machine is composed of the following elements; a constant light source, a photo cell in a scanning head, a rigid member to support the light source and the scanning head, ball bearing mounted slides in

orthogonal directions to allow positioning of the scanning head, motor driven lead screws to accomplish the scanning uniformly, suitable electrical controls for the scanning, a photographic negative positioning mechanism and a suitable frame for the support of the various parts. An assembly view is shown in Figure 16; a view showing location and configuration of the components is shown in Figure 12.

The mechanism to accomplish the linear transformation from location to time was carefully designed to accomodate precision synchronous timing motors. A reversible timing motor drives lead screws in each of the orthogonal directions to move a carriage at constant speed. The lead screws are 1/2 inch in diameter, 13 threads per inch, V-groove screws. In anticipation of periodic errors in the lead screw, the following nut was made of resilient low modulus teflon of long length. The carriage moves with respect to the average pitch of screw. The scanning head and light source move with respect to the fixed negative so that

$$Z = Kt \quad (4.4.1)$$

where K is constant to within 0.002 per cent of the maximum range of scan, which is 6 inches. The preceding results are determined from actual calibration.

Moire information is always obtained by scanning a negative in directions parallel and perpendicular to the lines of the reference grid. The negative is arranged in the platen of the machine with the lines of the reference grid parallel to one of the axes of the machine. A vacuum pump then holds the negative in position against a clear plastic portion of the platen. The light source is below the platen, the negative and the sensing head are above. Light passing through the negative activates a photo conductive cell which is a member of a Wheatstone bridge. The resulting electric signal is used in the further analysis of the information in the moire method of stress analysis.

By use of the photo-reader, positions of points on a photograph are determined to an error of ± 0.00012 inches or 0.002 per cent if full scale. If the magnification of the image is 1.0, then this represents the error in position on the model. If the magnification of the image is defined

$$m = \frac{\text{Image length}}{\text{Object length}} \quad (4.4.2)$$

then the error in reading positions is

$$\delta = \pm \frac{0.00012}{m} \quad (4.4.3)$$

The remaining errors in the data acquisition system are those of inaccurate film development to

proper gamma and voltage regulation. It is not difficult to maintain a relative error in photographic development of 5 per cent, which results in a maximum relative error of approximately 0.6 per cent in resistance of the photocell. The usual limits of $\frac{dR}{R}$ are 2 per cent with experience. These errors are removable by local averaging, as shown in Chapter III.

The voltages to the light source, a G. E. 10 long life bulb, and to the Wheatstone bridge are filtered and regulated to variations of 0.01 per cent.

The photocell is used to make relative intensity measurements for short time intervals. Thus, the effect of temperature and absolute resistance variations are negligible during the course of a single scan.

The linearity of the Wheatstone bridge voltage with resistance changes over the range normally experienced in scanning a moire photograph was found to be within 0.01 per cent by actual measurement.

4.5 The Data Reduction System

The method of data reduction is dependent on the sources of the errors in the complete system. These errors can be summarized for the various subsystems as follows:

- (1) The modeling system - a temperature reading error of 1 per cent maximum.

(2) Image forming system - an error in transducing displacement due to Poisson's affect a possible maximum in regions of concentrated loads of 1.5 per cent whether the model is evenly illuminated or not. A second error of unknown bounds arises as the result of uneven illumination and film spotting.

(3) Memory system - a possible error due to perspective if a second recording camera is used. This error may be very great and undetectable. Use of the difference method of analysis and the recording principle of the research will eliminate this error.

(4) Data acquisition system - it has been shown that these relative errors will be less than 1.0 per cent total and are removable by an averaging process.

Theoretically, in the absence of uneven illumination and film spots, the transformation from displacement to electric signal can be accomplished to a total relative error less than 3 per cent. For the thermal stress problem in the absence of concentrated loads this could be as low as 1 per cent. In this ideal case the electric voltage would be with e_K an arbitrary offset voltage from the Wheatstone bridge.

$$e(t) = e_0 + e_1 \cos \frac{2\pi u}{P} - e_K \quad (4.5.1)$$

corresponding to the displacement-intensity law.

$$I = I_0 + I_1 \cos \frac{2\pi u}{p} \quad (4.5.2)$$

A computer program could be devised to reduce this data and yield strain tensor components to whatever accuracy may be required. From this consideration it appears that an ideal lower bound on the error of an objective treatment of the data for the moire method would be slightly over 1 per cent.

Unfortunately it is almost impossible to eliminate some unevenness in the total illumination of a model because of differences in reflection or transmission functions of a material. The corresponding electric signal then is

$$e(x) = e_0(x) + e_1(x) \cos \frac{2\pi u}{p} + E'(x) - e_k \quad (4.5.3)$$

where $E'(x)$ is a local disturbance due to local defects in the film negative. This corresponds to an intensity distribution in the image perceived by the eye or a film of

$$I(x) = I_0(x) + I_1(x) \cos \frac{2\pi u}{p} + \varphi(x) + \dots \quad (4.5.4)$$

The remaining terms of statement (4.5.4) consist of elements removed by optical filtering by the reading device.

Several methods have been devised to reduce this data to strain components. The first of these methods is a semi-objective hand method.

The electric signal (4.5.3) is recorded on a Mosely X-Y plotter with maximum scale amplification. The recorded curve is shown in Figure 18. The first step is to locate the points of zero error, the background intensity. This is accomplished by locating the midpoints between a successive maximum and minimum. When all the mid-points have been determined a smooth curve is drawn connecting these points. This curve is the curve of the background intensity modulation. An envelope is drawn connecting all maximum and minimum points parallel to the background intensity curve. The maximum and minimum points being determined with respect to the background intensity curve. The modulation curve is directly related to the intensity function $I_0(z)$, and the envelopes to the amplitude function $I_1(z)$. True locations of the maxima and minima have been approached by finding these points with respect to the modulating function.

There may be several fringes which exhibit some minor amplitude variation with respect to the modulating function base line. It is assumed that for these fringes the greater number of points will be

free from error and that displacement information error is located principally at or near the extremes. The curves are normalized by plotting relative values of the function $\cos \theta$ at each ordinate; that is, at the extremes a new scale is established such that

$I_1(z) = 1.0$ and $I_0(z) = 0$. Curves corresponding to 0.866, 0.707 and 0.500 are drawn parallel to the base line. Intersection of these lines and the data curve represent points of $\theta = 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}$ and $\frac{\pi}{2}$ in each

$\frac{\delta}{4}$. Projection of the location of these points on to the ordinate of a curve which will have fractional units of p as the abscissa will construct a curve of the displacement function U . Each cycle of the intensity curve from one maximum to the next will yield 15 points on the curve of U . Most of these are from regions of high accuracy. Carefully drawing a smooth curve through the average of this set of points giving equal weight to each point will greatly reduce the remaining imaging errors. Extrapolating the curve through small intervals containing extraneous information caused by local defects in the film will eliminate the image error in these regions. Graphic differentiation is used to deduce the strain components from the curve of U .

This method presents the most accurate method of hand reduction of moire data. The method uses the four improvements in the moire method of stress analysis; that is, a photo-reading device to transform from intensities to voltage, the intensity-displacement law to use low error information, a local averaging process to use compensating errors in the image and mismatch to reduce amplitude modulation errors. However, the method is not completely objective.

In the absence of a Dymec 2013R system, a substitute method was found to use the equipment available. The RW 300 computer has an analog to digital capability with punched tape output. However, the accuracy is plus or minus one digit in four with only one range, plus zero to 10,000 volts. To exceed the capability of hand analysis, the photo-reader information must be amplified and confined to this range.

Several units were assembled to accomplish the demonstration of the feasibility of analysing moire data by completely objective means. These include a narrow band pass electronic filter, which effectively removes most of the effect of uneven illumination and much of the high frequency noise due to local film defects. An analog computer is used to amplify the photo-reading signal and convert it to absolute value so

that the total range of the RW 300 computer can be used. The arrangement of the equipment including the photo-reader, the filter, analog computer and the RW 300 are shown in Figure 19. The IBM 709 program was modified to accept the information as presented by the RW 300. It will be noted that the accuracy of the computing system is independent of the problem and may be tested by inserting a known electric signal.

An analog computer program to accomplish the same analysis was devised. To utilize the analog computer for this analysis the photo-reading device must be altered. A second reading head must be installed to read the fringe with what is known as a transport delay. The output from the photo-reader would be two voltages from the same points but separated in time by an interval. Call these signals $Y(t)$ and $Y(t + \Delta t)$. The schematic of the analog solution is shown in Figure 20. The operational amplifier, shown as 39, serves as a buffer and isolation amplifier for $Y(t)$. The circuit from 30 including the amplifier 42, 54 and 43 convert $Y(t)$ to $A|Y(t)|$. The amplifier 34 and 41 act as local memory, constantly storing the present value $A|Y(t)|$ until $A|Y(t)|_{\max}$ is attained. The comparator to accomplish this is marked 5 on Figure 20.

The local memory begins to track the signal $|A Y(t)|$ until $|A Y(t)|$ approaches zero. This is accomplished by the comparator 7. The local value of $|A Y(t)|_{\max}$ is transferred to another local memory composed of amplifiers 35 and 44 by the command from the second input signal $|A Y(t+\tau)|$. The conversion from $Y(t + \tau)$ to $|A Y(T + \tau)|$ is accomplished by the identical amplifier channels 45, 46, 37 and 10. The signal $|A Y(t + \tau)|$ is divided by $|A Y(t)|$ to yield the normalized signal $|Y(T + \tau)|$. The modulation errors of the image are reduced in a manner very similar to that purposed for the optimum digital program.

The solution for U is accomplished by having a repetitively operating sine-cosine generator in conjunction with a generated ramp function. In this way $|\sin \Theta|$ and Θ are generated to the same time base, which is very much faster than the real time or signal time base. Comparison of $|Y(t + \tau)|$ and $|S \sin \Theta|$ in a comparator causes the generation of this function Θ and this is the same as U . Local averaging of U is accomplished by a passive R-C network of time constant proportional to δ . The resulting well behaved analog voltage can be differentiated by analog means to yield a voltage proportional to the strains.

The preceding system has been assembled and tried. The input signals $Y(r)$ and $Y(t + \tau)$ were simulated. Curves of U were obtained. The best accuracies that could be obtained from the system were in the order of a relative error of 0.6 per cent. These errors stem from the finite transfer time through the circuits and from the use of many comparators. The transfer delays were caused by the use of servo-multiplier-divider mechanisms to accomplish the division required. The comparators in which two voltages are compared were also of mechanical type, that is, electromagnetic relays, and required a finite though small current flow. Both of these error sources could be greatly improved by the use of all electronic divider and comparator circuits.

The problem of data reduction is resolved into a problem of separating desired signal from noise. When initial mismatch is used, the desired signal contains displacement information at discrete spatial frequencies. The subsequent deformation of the model causes the information to be spread in small bands centered on the mismatch frequency. The background illumination function

can be considered to be made up of infinite summation of sinusoids, each of wavelength of some integer multiple of the model size. If large numbers of initial fringes are used, the background noise frequency spectrum is far removed from the signal noise. This fact makes filtering either electrically or optically possible. Local defects cannot be adequately filtered and at those points the information is in error. The possibility of optical filtering has been discussed and an electronic filter has been mentioned. The electronic filter used is a bandpass type. The pass band is completely adjustable from 0.02 to 2000 cycles per second.

When the electronic filter is used the center frequency is chosen by the fringe spacing on the negative to be scanned and the photo-reader speed of scanning. Thus, if twenty seconds is required to scan one fringe, the center frequency is 0.05 cps. The lower cutoff frequency is set one octave down or 0.025 cps, and the upper is set at 0.10 cps. Thus, assuming a full two-octaves band pass which will easily contain the displacement information, the speed of the photo-reader scan and the lowest frequency available on the filter places an upper limit on initial fringe spacing, which is 0.20 inches on a negative.

4.6 A Specific Discussion of the Moire System

Sources of error in each subsystem of the moire method of stress analysis have been discussed in the preceding sections. A general outline of the characteristics of these errors was used to explain the sources of the data reduction method. Four improvements in the moire method were outlined. In this section a specific discussion related to a specific problem will be presented. The intent is to demonstrate the interrelationship of the errors possible and to defend the need for complete objectivity.

The model to be discussed is a tension sample experiencing a uniform strain ϵ . It will be assumed that the errors in technique have largely been avoided; that is, the reference grid is without error, no lack of parallelism between model surface and image plane exist and that lines are imprinted in a perfect fashion upon the model. It will be assumed that a usable negative or image is available for each case. It will also be assumed in each case that the fringe spacing due to ϵ is 1,000 inches and $p = 1/300$ inches.

Case 1. No initial mismatch exists and the model illumination is uniform. Data acquisition is accomplished by eye and data reduction by hand.

The contrast between the intensities at two points is defined

$$\gamma = \frac{I_1 - I_2}{I_1} \quad \text{WHEN } I_1 > I_2 \quad (4.6.1)$$

A sinusoidal intensity distribution between fringes is assumed and the points of interest are those near the maximum intensity. Thus

$$\gamma = \frac{I_1 - I_2 \cos \theta}{I_1} \quad (4.6.2)$$

A normal eye has a limiting sensitivity to γ as defined by (3.7.1) given by reference (48). The limit for a normal eye is $\gamma = 0.02$. It is assumed that extraordinary eyes exist for which the limit may be $\gamma = 0.002$. For the ordinary eye, then the limiting angle θ for which a distinction can be made then is

$$1 - \cos \theta = 0.02 \quad (4.6.3)$$

or

$$\theta = \pm 11^\circ \quad (4.6.4)$$

this is a relative error in detecting maxima of

$$\pm \frac{11}{360} = \pm 0.03 \quad \text{or } 3\% \quad (4.6.5)$$

For the extraordinary eye the corresponding relative error is

$$\pm \frac{35}{360} = \pm 1.0\% \quad (4.6.6)$$

The extraordinary eye could thus detect 0.01 fringe spacing as suggested by Sciammarella (40).

Linear measurements can be made upon the film with the help of a 10X magnifying glass to ± 0.001 inches. This corresponds to a relative error in fringe location of

$$\pm \frac{0.001}{1.000} = \pm 0.001 \text{ or } 0.1\% \quad (4.6.7)$$

The curve of U is constructed using a convenient scale for p . The limit of resolution for the unaided eye is a spot 0.010 inches in diameter. The scale of p is chosen so that $p = 10.00$ inches. This can be located with a relative error of 0.1 per cent on the paper. For the same reason a magnification of linear scale of distance is established so that the relative error in plotting is 0.1 per cent. The point representing the maximum point is now located with definite bounds on the analysis paper. Vertically, it is located with an accuracy of ± 0.1 per cent, horizontally to ± 1.2 per cent, when the extraordinary eye is used.

Since p is fixed by the reference grid spacing the only limitation in sensitivity of the method is the degree of accuracy with which the points can be located. In this case the sensitivity in displacements and strains is

$$\Delta U = U^* = 0.012 p \quad (4.6.8)$$

The strain ε may be determined to an accuracy equal to the accuracy of the slope of the curve of U and an accuracy in determining what that slope is. The accuracy of the slope of the curve can be shown to be

$$\left| \frac{d\varepsilon}{\varepsilon} \right| = \left| \frac{dU}{U} \right| + \left| \frac{dz}{z} \right| \quad (4.6.9)$$

The accuracy of determining what the slope is by eye is thought to be on the order of the ability to measure linear distances, approximately 0.1 per cent relative error. Thus for the simple example

$$\left| \frac{d\varepsilon}{\varepsilon} \right| = 1.4\% \quad (4.6.10)$$

Note the extreme loss of sensitivity and accuracy that is possible if a normal eye is used.

$$U^* = 0.03 P \quad (4.6.11)$$

and

$$\left| \frac{d\varepsilon}{\varepsilon} \right| = 3\% \quad (4.6.12)$$

Case II. No initial mismatch exists and uniformly varying background illumination is assumed. Data acquisition is by eye and data reduction by hand.

As explained in the previous section the darkest and lightest portion of the fringes will be misplaced in the recording. If K is assumed to be 1.0

the relative error in the location of the maxima can be found.

$$\Delta z = \frac{K}{2\pi} = \frac{1.0}{2\pi} \quad (4.6.13)$$

and

$$\frac{\Delta z}{z} = \frac{1}{2\pi} \text{ or } 16\% \quad (4.6.14)$$

with the extraordinary eye a further error exists of 1.0 per cent.

Points on the curve of U are known with relative errors of 1.1 per cent vertically and 17.2 per cent horizontally.

The strain and displacement sensitivity have been reduced to

$$U^* = 0.172 P \quad (4.6.15)$$

The relative error in strains is then

$$\left| \frac{d\epsilon}{\epsilon} \right| = 17.4\% \quad (4.6.16)$$

Case III. Initial mismatch of $\beta = 4\epsilon$ is used; uniformly varying background exists. Data acquisition is by eye and data reduction is by hand. For this case of illumination, comparable to Case II, $K = 0.2$.

$$\Delta z = \frac{0.2}{2\pi} \quad (4.6.17)$$

The relative error depends on the fringe number, For this case

$$\frac{\Delta Z}{Z} = \frac{0.2}{2\pi} \cdot \frac{1}{(-1)^n 0.1n} \quad n = 1, 2, 3, \dots \quad (4.6.18)$$

The relative error in location of the fifth fringe maximum is 3.2 per cent. The error of the extraordinary eye and the plotting error must be added to each of these relative errors of the points. Because of the alternating sign of the error a smooth curve drawn through the points giving equal weight to each point will have an error much smaller than the minimum. Also, in this example, there are more positive errors than negative. The curves will be displaced from 1 per cent to 2 per cent. Thus the displacement sensitivity will be

$$U^* \cong 0.02 p \quad (4.6.19)$$

and

$$\left| \frac{d\varepsilon}{\varepsilon} \right| = 2.2 \% \quad (4.6.20)$$

Here the use of mismatch and local averaging has brought the sensitivity and the accuracy of the moire method within reasonable bounds.

Case IV. Initial mismatch of $\beta = 4\varepsilon$, uniformly varying the intensity function. Data acquisition by photo-reader and data reduction by hand.

For the photo-reader no limiting γ exists but two smaller errors due to improper film development and voltage stabilization of the bridges do exist. The latter error has little effect on the analysis of the moire data, for the voltage fluctuations in the time interval of scanning will be less than 0.01 per cent with a long term stability of 0.1 per cent. A relative error of 2 per cent in film gamma causes a maximum error of 0.25 per cent in places of high intensity and 0.10 per cent in places of low intensity. The error causes no ambiguity in determining points of relative maxima and minima but does introduce a small error in application of the displacement optic law.

The output of the Wheatstone bridge is recorded on a Moseley XY plotter operating as a strip chart recorder. The relative error in the position of the recording pen in the Y direction is a reflection of the linearity of recorder, which is ± 0.1 per cent. The relative error in the X location of pen with respect to the strip chart is measured by the linearity of the strip chart mechanism which is 0.01 per cent from the manufacturer's specifications. The X-direction linearity of the photo-reader was determined by measurement to be less than 0.01 per cent. Thus X positions of points on

the recorder graph of intensities are known to within a relative error of 0.02 per cent. Points of minimum intensities are located in the Y direction to a relative error of 0.20 per cent and points of maximum intensities to errors of 0.35 per cent. Points on the background intensity curves will have errors of approximately 0.25 per cent. The errors in the X direction are the important ones.

Notice that when images with no background illumination error are scanned, the resultant maxima and minima are located to a relative error due to the X-Y plotter of 0.1 per cent, a tenfold advantage over the extraordinary eye. Also that the location of points are determined to an accuracy of 0.00012 inches or a linearity of 0.02 per cent. This again represents a tenfold increase of sensitivity over the eye aided by a 10X magnifying glass.

For the specific example being examined the same image errors are recorded in the film and plotted as intensity versus location of the X-Y plot.

The advantage of the method is easily seen to be that relative maxima and minima with respect to a fixed datum need not be accepted as with scanning by eye. The true maxima and minima may be determined

relative to the background intensity curve. Considering that each point of the intensity curve has a Y direction error of 0.27 per cent in the region of the background curve; it is reasonable to expect that this latter curve can be established with a relative error of 0.20 per cent in the vertical direction. On the curve of intensities point of maxima and minima can be determined to an accuracy of ± 0.020 inches in a fringe spacing of 0.21 inches. This represents a relative error of 0.1 per cent.

Thus in a curve of U determined by plotting, points of maxima and minima are known with a relative error in the X direction of 0.12 per cent. Again, since the ordinates are multiples of p , only plotting errors are involved. So the Y direction relative errors are ± 0.1 per cent as before. The displacement sensitivity using only points of maxima and minima is

$$U^* = 0.0012 P \quad (4.6.21)$$

When the displacement optic law is used, many points of the curve of U may be obtained. Local averaging by graphically drawing the best fit removes the photo cell error and the displacement sensitivity is again,

$$U^* = 0.0012 P \quad (4.6.22)$$

at every point. The corresponding relative error in strains being

$$\left| \frac{d\varepsilon}{\varepsilon} \right| = 0.14 \% \quad (4.6.23)$$

or as in Case II adding manual slope sensing error 0.1 per cent

$$\left| \frac{d\varepsilon}{\varepsilon} \right| = 0.24 \% \quad (4.6.24)$$

A tenfold increase in accuracy at every point by the use of the photo-reader, background illumination removal, the displacement-optic law and an averaging process.

Case V. All conditions equivalent to those of Case IV except data reduction by computer in the optimum fashion.

The analog signal from the photo-reader is amplified directly and introduced into an analog to digital converter and the output is punched tape. The process from punched tape to IBM 709 memory is without significant error. With the Dymec 2013B system the resolution is ± 1.0 mv in 10 volts. The time base linearity of the IBM 709 is on the order of 10^{-5} ; therefore, no new errors of significant amounts can be introduced from this source. The relative errors in the information stored in the memory of the computer

can be referred to a hypothetical plot on the XY plane similar to the plot of previously considered.

After including a relative error from the amplifier (0.01 per cent) and a resolution error, the error in the stored information is

$$\left| \frac{dy}{y} \right| = 0.02\% + \text{Photocell error} \quad (4.6.25)$$

$$\left| \frac{dx}{x} \right| = 0.02\% + 0.01\% \approx 0.03\% \quad (4.6.26)$$

Ordinarily an additional error will be introduced by assuming a chordal approximation to the background intensity curve. The error will be of small magnitude if the chordal approximation is taken for each one-half cycle. It will also be of opposite sign for alternate one-half cycles. This implies that local averaging of the curve of U over one cycle will greatly reduce the error. In further discussions this error will be included with the photocell error as one to be eliminated by averaging. For the specific example included here, no additional errors are involved in the theory of taking a chordal approximation.

For most of the computer operations involved the resulting errors will be infinitely small. However, it is easy to see that a possible error of 0.005 per cent in $\frac{dy}{y}$ may occur in the selection of points on the background intensity curve.

Another 0.005 per cent error in $\frac{dy}{y}$ may enter through the normalization process in subsequent roundoff of division for final plotting purposes. Again consider $y = y(\theta)$ to separate the source of relative errors of U . In the computer program

$$\theta = \arctan \frac{\bar{y}}{\sqrt{1-\bar{y}^2}} \quad (4.6.27)$$

Here $\bar{y}$ is the normalized y and disregarding infinitesimal of higher order

$$\frac{d\bar{y}}{\bar{y}} = \frac{dy}{y} \quad (4.6.28)$$

The relative error in θ is

$$\frac{d\theta}{\theta} = \left[\frac{1}{\sqrt{1+\frac{\bar{y}}{\sqrt{1-\bar{y}^2}}}} \cdot \frac{\bar{y}^2 + \bar{y} - 1}{(1-\bar{y}^2)^{3/2}} \cdot \frac{\bar{y}}{\arctan \frac{\bar{y}}{\sqrt{1-\bar{y}^2}}} \right] \frac{d\bar{y}}{\bar{y}} \quad (4.6.29)$$

or

$$\left| \frac{d\theta}{\theta} \right|_{\text{MAX}} = \left| \frac{d\bar{y}}{\bar{y}} \right|_{\text{MAX}} \quad (4.6.30)$$

The limit of the bracketed function as $\bar{y} \rightarrow 0$ is 1.0 and as $\bar{y} \rightarrow 1.0$ the limit is 0. Therefore, the maximum relative error in the displacement U is determined from (3.7.36) and considering (3.7.40) and (3.7.35) plus an additional 0.01 per cent for the computer errors mentioned,

$$\frac{dU}{U} = 0.03\% + 0.01\% + 0.02\% + \quad (4.6.31)$$

Photocell error

Doubly integrating over an interval of one wave length assures the removal of the photocell errors. Then the displacement sensitivity is

$$U^* = 0.0006P \quad (4.6.32)$$

and by the criteria used in the other cases

$$\left| \frac{d\varepsilon}{\varepsilon} \right| = 0.087 \quad (4.6.33)$$

Differentiation of the curve of U by finite differences is limited only the nature of the remaining errors in the function U . If the error distribution is Gaussian, local values will be in error, but the averaging process over the interval of one wave length of the signal will minimize the errors in the derivative.

Case VI. All conditions the same as Case V except the RW 300 computer is used as an analog to digital converter.

From the results of Case V it can be seen that errors in the input to the computer are reflected directly in output errors. The RW 300 has a resolution of 0.1 per cent in y and a time base linearity which far exceeds that of the photo-reader. The amplification and absolute value are determined by analog operational amplifiers capable of 0.01 per cent linearity. Therefore, the information in the memory of the IBM 709 corresponding to (4.6.25) and (4.6.26) is

$$\frac{dy}{y} = 0.1\% + 0.02\% + \text{P.C. error} \quad (4.6.34)$$

$$\frac{dx}{x} = 0.01\% \quad (4.6.35)$$

The corresponding relative error in displacement is then

$$\frac{dU}{U} = 0.12\% + 0.01\% + \text{P.C. error} \quad (4.6.36)$$

After the double smoothing to remove photocell error the displacement sensitivity is

$$U^* = 0.0013 P \quad (4.6.37)$$

and

$$\left| \frac{dE}{E} \right| = 0.15\% \quad (4.6.38)$$

The results of Case VI show that complete objectivity by substitute means does not in general greatly increase the accuracy and sensitivity of the method over that attainable by hand analysis as in Case IV. The basic difficulty in Case VI is that the sensitivity of the RW 300 is the same as the Moseley X-Y plotter and the eye.

This specific example points out the advantage to be gained by use of the following improvements in the moire method:

(1) The use of mismatch to allow a base to detect background illumination variations.

(2) The use of a photoreader to transduce the intensity variations and locations.

(3) The subsequent removal of background illumination.

(4) The use of the displacement-intensity law to allow continuous determination of the curve of U .

(5) The use of local averaging to remove any errors introduced by the analysis method.

(6) The use of a computer to perform all measurements and to make the analysis objective.

The foregoing discussion of the effect of the relative errors is not intended to be realistic. It is intended to display a consistent analysis, whereby conclusions concerning the desirability of the various methods of computations can be drawn. The sensitivities shown can be thought of as lower bounds in each case.

CHAPTER V

APPLICATIONS

5.1 Application Number One--A Tension Sample

The model used in this test was a tension sample prepared from a sheet of Hysol 8705 plastic. A grid of 300 lines per inch had been printed on the sheet using a photoengraver's albumenoid base coating.

The purpose of this test was to develop and demonstrate the new mode of operation with the moire method of stress analysis. Figure 21 is a composite photo montage showing in the inset, a reduced over-all view of the model. A greatly enlarged section of the model is shown with the corresponding reduced scale photo of the photo-reader record. The first attempt at analysis by hand is shown on the photograph. The corresponding curve of U is shown superimposed to the same scale. To estimate the scale adjustments, the fringe spacing on the model is on the order of 0.20 inches, while the entire photo-reader trace was about 30 inches. Thus a magnification of linear scale exists, which is about twenty times.

Figure 22 shows the completely objective reduction of data using the R.W. 300 and the I.B.M. 709 computers. Approximately the same linear scale magnification exists and the same portions of the film were scanned. The curves (A) at the bottom of Figure 22 are the absolute value of the photo-reader curves shown in Figure 21, with most of the uneven background illumination removed. Curves (B) represent the normalized curves. The curve of u is the diagonal sloping curve (C) while the derivative curve (D) is the plot of local strain components, $\frac{\partial u}{\partial x}$. The uncorrected curves of u and $\frac{\partial u}{\partial x}$ are shown before local averages are taken.

This test represents the first time that strain components at a point have been reduced in a completely objective fashion by the moire method. As in any first effort, the results are not perfect.

In the hand analysis of the data, only one value for the $\cos \Theta$ was chosen, that being 0.707. This results in eight points on the curve of u with abscissa increments of $\frac{\pi}{4}$. The data in the curve of u for Figure 21 and correspondingly, the uncorrected curve of u drawn by the computer shows the photocell and normalization errors. It shows how these errors are

removed by local averaging. In the curve of $\frac{\partial u}{\partial x}$ drawn by the computer, an additional error, which has not been discussed, is evident. It is manifest in the undulations of the derivative curve at the end of the plot. The exact source of the error is unknown at this time. Speculations as to possible causes, not included in the previous analysis, include consideration of the transformation from chemical energy to electrical energy in the data acquisition system. Here a vectorial addition of amplitude functions, instead of a scalar sum of intensity functions, may occur in the optics of the photocell. Photocells also admit secondary currents which may distort the signal. There is also a transient characteristic of an electronic filter, which is called the settling time. During the settling time the desired signal is distorted in amplitude. This distortion transient decays, and the filter then has a linearity of less than 0.01 per cent. This undesirable characteristic of an electronic filter is inherent to the machine by virtue of the finite transit times of electrons, in an electronic circuit. In the optimum computer method, no electronic filter would be required.

In spite of the critical observations just mentioned, the results of test one disclose many constructive improvements to the moire method of stress analysis.

1. The superior resolution of the photocell over the eye in intensity measurements is clearly demonstrated.

2. The superiority of linear scale amplification by machine measurements is clearly demonstrated.

3. The existence of a background intensity error, which cannot be detected by the eye, is clearly demonstrated.

4. Errors arising from known sources and of magnitude that may be controlled have been substituted for unknown errors and this is clearly demonstrated.

5. The displacement intensity law yields an infinite number of points to the curve of U with errors of known bounds. This has been clearly demonstrated.

6. Complete machine reduction of data can be achieved. This has been demonstrated.

7. With the method of analysis by use of the displacement-intensity law and a photo-reader, the moire method becomes the first stress analysis method to measure displacement at a point on a surface. This has been demonstrated.

Because of the one-dimensional characteristic of the problem of test one, estimates of sensitivity and accuracy were not attempted.

5.2 Test Number Two--a Ring Loaded in Diametral Compression

The model for test number two was a ring 4.00 inches in O.D. and 2.00 inches in I.D., cut from the sheet of Hysol described in test one. Diametral loading was applied by a suitable loading frame. The illumination was by transmission from a diffuse source. Grids of 300 lines per inch were used. In Figure 23 the loaded and unloaded state of the ring are shown.

An initial mismatch in grid pitches of 3 per cent was used to allow determination of the background intensity curve. Analysis was by using the described hand technique. The results are shown in comparison with the theoretical solution in Figure 24. The minimum strains detectable were 20×10^{-6} inches/inch. This corresponds to a strain sensitivity of

$$\epsilon^* = 0.006 p \quad (5.2.1)$$

which is a ten-fold increase in sensitivity over the previous methods of analysis. This represents an improvement on the theoretical limit of the hand

method of analysis, which indicates that in the local averaging process, accuracy is improved and the actual theoretical limit is less than that shown in Section III.7. No computer analysis of this data was attempted so a comparison similar to case one cannot be shown. However, between test one and test two the method has been tested for cases of illumination by transmission where uneven illumination is not usually a great problem.

5.3 Test Number Three--Thermal Stress in a Ring

In this test a ring of low carbon steel was used. The dimensions were 5.50 O.D. and 0.500 thickness. The tests were conducted in the manner described in a previous paper (2). That is, the lines were imprinted in a model and a fused silica master by use of Kodak photo resist and mechanical contact between model and reference master was by the use of vacuum.

Figure 25 shows the initial mismatch of 1 per cent before heating and the moire fringes produced by heating.

Results of the test are shown in Figures 26 and 27. These show the close agreement between theoretical stresses and experimentally determined stresses as previously reported.

The information obtained by using crossed grids of 300 lines per inch with the photo-reader and hand analysis of the data was more accurate than that obtained previously by using 1,000 lines per inch.

5.4 Test Number Four

This test was the first in the series to accomplish the over-all goal of the thermal stress program. Rings made from number 304 stainless steel were heated by induction heating as previously described (see Chapter IV). One ring of this set was chosen to furnish the first data quantitatively reduced by the new moire system.

A Dymec 2010B data acquisition system was obtained on a loan basis. This system was integrated into the photo-reading system to provide computer compatible information. That is, the electric signal from the photo-reader was converted to digital form and recorded on punched tape by the Dymec unit. The arrangement is shown in Figure 28.

The Dymec unit consists of an integrating voltmeter, a parallel to serial converter and a punched tape output. In use in this test, the photo-electric

signal was integrated over a 0.1 second span of time while the sampling rate was approximately five words a second, resulting in a reading every 0.00149 inch on the model. The digital voltmeter reads to six places with a minimum sensitivity of 0.1 millivolt. The particular ring selected was 5.50 inches O.D. and 2.50 inches I.D. The initial mismatch pattern of approximately 5 fringes per inch and the final heated pattern at 1,120°F were analysed. Photographs of these two patterns are shown as Figures 29 and 30, respectively.

The negatives were scanned in the radial direction and in the tangential direction to obtain ϵ_r and ϵ_θ along a radius. The elastic solution using average values of E , ν , and α were computed and are shown in Figure 31. The data was obtained on punched tape, converted to I.B.M. cards and processed in the I.B.M. 709 computer, using the program listed in Appendix B. The computer reduced data consists of printed sheets and plotted curves. The plotted curves are used as a monitor of the system to enable the observer to detect any malfunctions, while the actual results are obtained from the numbers in the computer results. Figures 32 and 33 show the computer drawn curves of the initial and final conditions respectively. Figure 34 shows a sample of the printed data.

Shown in the figures is the corrected data with background intensity variations removed. The normalized curves are also displayed as previously described. Curves of u and u averaged by integrating over one fringe spacing, are shown superimposed as the diagonal curve. The derivative $\frac{\partial u}{\partial x}$ is shown multiplied by a factor of five in the first horizontal curves and greatly magnified in the curve shown at the top of the page. The data is quantitatively reduced as follows. Referring to Figure 34, the first column is the number of the point; the second column is the absolute value of the moire information; the third column is the normalized data; and the last column is the strain component $\frac{\partial u}{\partial x}$. The total distance on the model is divided by the number of points listed and is the distance represented by each reading. In this case, it is 0.00149 inch. Between successive maxima, indicated by 1.0000 in column 3, a displacement of $1/600$ inches by 0.00149 n , where n is the number of points between maxima, yields an average strain between the two maxima. Comparison with the average value of the strain indicated for the interval establishes a conversion constant. This procedure may be accomplished most accurately and easily on the data of the undeformed

ring. Checks by repeating the procedure anywhere in the data should confirm the established constant of proportionality. Thus, the differences between the last columns are multiplied by the constant and plotted as strains, as on Figure 35. While there is considerable scatter at points where the undulation in the curves of strain exist, the multiplicity of points allows the plotting of the true strain component curve. Comparison of the experimentally determined curves of strain with the calculated curves of strain using the elastic solution with constant average values of E , ν , and α is shown in Figure 36.

When the thermal dilatation $\alpha \Delta T$ is subtracted, the stresses can be calculated at each point by the generalized stress-strain relations. The results are shown in Figure 37 compared to the elastic solution. It will be noted that in general the stresses are lower than would be predicted on the basis of constant physical properties.

That a solution of this particular thermal problem has been found is shown by the equal areas under the opposite sections of the curves of $\sigma \epsilon$. Compatibility and equilibrium have been identically satisfied.

The sensitivity of the method can now be discussed in terms of numbers. Each displacement of 1 pitch was divided into 623 parts. Therefore, with the system used, the displacement sensitivity is

$$U^* = \frac{1}{623} P = 0.0016P \quad (5.4.1)$$

which compares with the maximum displacement sensitivity of the best eye.

$$U^* = 0.012 P \quad (5.4.2)$$

or to the maximum theoretical limit of

$$U^* = 0.0006 P \quad (5.4.3)$$

The practical limit to sensitivity and accuracy is in the signal-to-noise ratio. As measured prior to this test, a fixed noise signal of approximately plus and minus 2 millivolts was in contrast to a signal on the order of 1 volt. This signal-to-noise ratio appeared in the final derivative as a plus and minus 5 units in 1,510.

This corresponds to a relative error

$$\left| \frac{dE}{E} \right| = \frac{\pm 5}{1510} = 0.33 \% \quad (5.4.4)$$

which is quite high in comparison to the theoretically expected value from (4.6.33)

$$\left| \frac{dE}{E} \right| = 0.08 \% \quad (5.4.5)$$

but much lower than that which can be accomplished with the extraordinary eye, which is, from (4.6.10)

$$\left| \frac{d\varepsilon}{\varepsilon} \right| = 1.4\% \quad (5.4.6)$$

In addition, undulations appear in the curve of the derivative. These increase the scatter of points of the plotted curve of $\frac{\partial u}{\partial x}$ due to thermal stress. The multitude of points of low error easily define the curve, which corresponds to the concept of the continuum. The undulations need further study since the probable origins are:

1. Local film noise which perturbrates the curve of ε as mentioned earlier, or
2. Actual model surface phenomena, which are not accounted for in the optical analysis, or
3. The true behavior of the polycrystalline material, inasmuch as this is the first observation of displacement at a point.

The fact that the undulations are not present when test data, consisting of a long wavelength noise superimposed on a pure sine wave is analyzed indicate the source as the model, negative, or photocell. Observations of the photocell tend to eliminate this unit as the source.

5.5 Test Number Five

During the progress of the series of tests to accomplish the major goal of this research, a ring was heated in an attempt to achieve thermal plasticity. Visual examination of the pattern did not indicate that a plastic condition had been achieved. It was decided to analyze this set of data with the photo-reader-computer combination. The initial and heated pictures are shown in Figures 38 and 39, respectively. The temperature of the ring of Figure 39 is 1,580°F on the outside and 920°F on the inside circumference.

Analysis of the data exactly as in test four indicates a strain which cannot be explained by elastic deformation. The computer curves are shown in Figure 40 and the plotted strains in Figure 41 compared to the elastic strain, calculated for the problem.

Once the elastic plastic region was ascertained from the curve of strain, the region can be detected by eye in the photograph. Notice the inner 1/3 of the radius in Figure 39. A more complete analysis of this ring will follow in the subsequent reports of this program.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

The problem of this research has been, as stated in Chapter I:

1. To develop a completely objective procedure for two-dimensional experimental stress analysis from the moire method and

2. To develop the technology of using the objective moire method in thermal stress problems, including a question concerning this applicability of the Duhamel Neumann law in experimental stress analysis at high temperatures.

Chapter II has indicated the general state of knowledge of these problems and has proposed a possible new approach.

Chapter III has been used to develop, in generality, the concepts of the experimental method used.

Chapter IV has discussed the specific developments of the moire method of stress analysis to accomplish the solutions to the problems as stated above.

The results of the research are presented in Chapter V. Test number one clearly indicated that the first objective of the research had been attained. However, test number four attests that both objectives of this research have been attained. This is demonstrated by the analysis, in a completely objective manner, of a moire fringe pattern taken from a ring heated to 1,580°F.

Sources of errors in the moire method that were previously unknown have been disclosed. Corrective procedures have been devised. A precision photo-reader and a data reduction system have been built and used in a unique, useful manner.

This research has initiated a new era in experimental stress analysis. For the first time a statement of accuracy and sensitivity of a particular experimental stress analysis method can be assigned to the method and not to the operator.

Specifically, the conclusions drawn from this research can be stated as:

1. The moire method stress analysis can be reduced to a completely objective test for the determination of two-dimensional strain fields.

2. The moire method can be used in the general class of thermal stress problems to temperatures of 1,700°F with certainty and to higher temperatures, including convective heat transfer boundary conditions, with minor development.

3. Considering the results of test number four, where the variations in E , ν , and α were slight, and the favorable comparison of the results with the analytical elastic solution to the same problem, it must be concluded that the basis of the Duhamel Neumann law is valid, that is, that total deformation is composed of a thermal dilatation and a deformation due to thermal stress. The constants in the expansion shown in Chapter II must be used as functions of the absolute temperature T .

4. It may also be concluded from test four that results of analytical computation, using the elastic solution to the thermal stress problem of the ring will be conservative, that is, these computations yield resulting stresses which are greater than those arising in 304 stainless steel at the same temperature.

Recommendations for future research would specifically include the following considerations of the moire method:

1. An investigation to determine the source of the undulations in the curve of $\frac{\partial u}{\partial x}$ as produced by the computer. A possibility exists that these represent true behavior of the polycrystalline material, a question which must be resolved to serve as a guide for the concept of isotropy and homogeneity in analytical work.

2. Techniques need to be developed to improve the contrast in the image or generally to alter the gamma of an existing photograph for improved signal-to-noise ratio. To clarify this statement, it must be admitted that optical filtering of the first harmonics of the intensity function is more desirable than electronic filtering. Perhaps a method can be found to filter optically all negatives before reading.

3. A great effort must be made to remove the "art" from the "science" of experimental stress analysis before a universal tool will emerge. Specifically, this means that grids must be available to be attached to models without a great knowledge of the photo-engraver's art.

At the conception of this program of research it was realized that there were many areas of uncertainty concerning thermal stress and that no

tools existed to study these areas experimentally. Now the tool exists; quite general considerations of the thermal stress problem were made to establish this tool. Subsequently, it becomes necessary to define specific areas of uncertainty and to investigate each in a logical fashion. This is the recommendation concerning thermal stress.

APPENDIX A

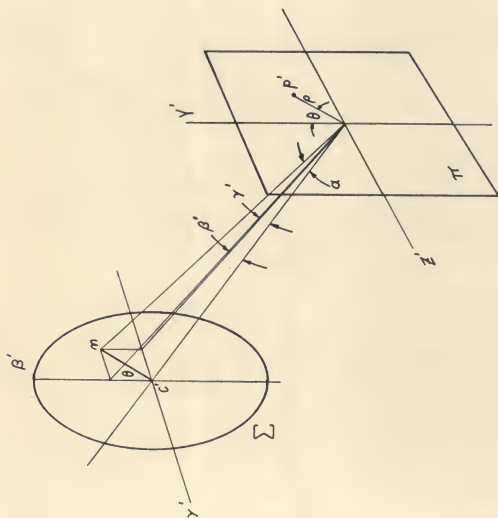


Figure 2. 3-D Sketch of Imaging System

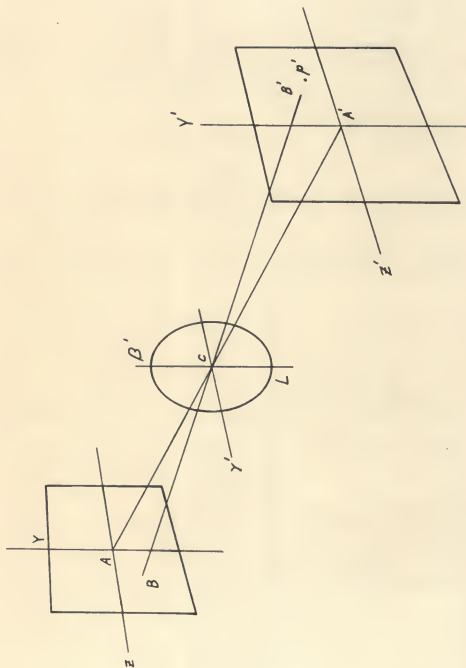


Figure 3. New Notation for Imaging an Extended Object

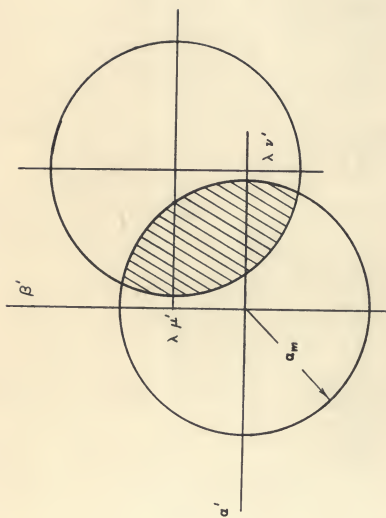


Figure 4. Area of Integration of g

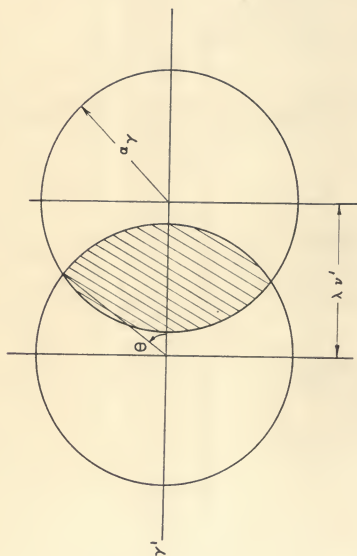


Figure 5. Alternate of Figure 4

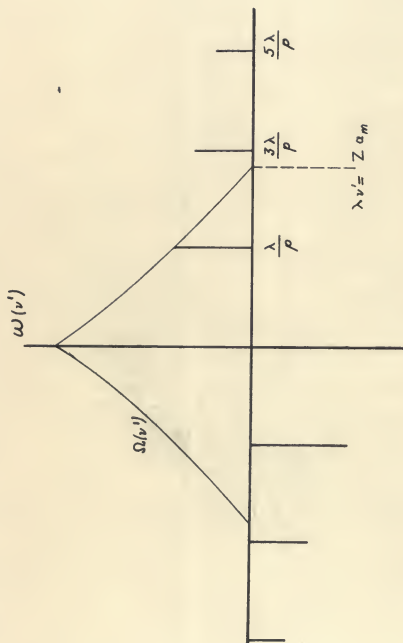


Figure 6. Filtering Action of the Lens

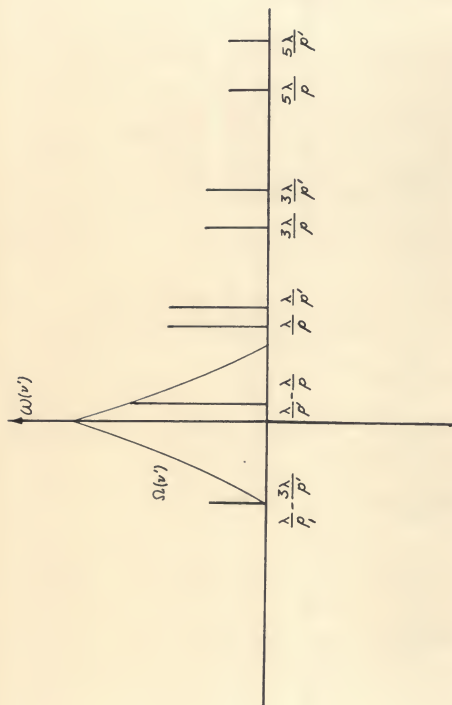


Figure 7. Filtering of Spatial Frequencies by Subsequent Lens

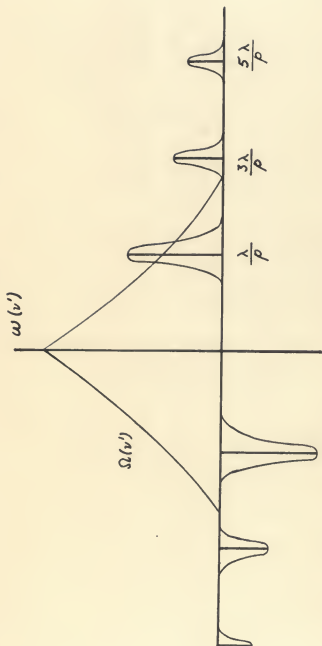
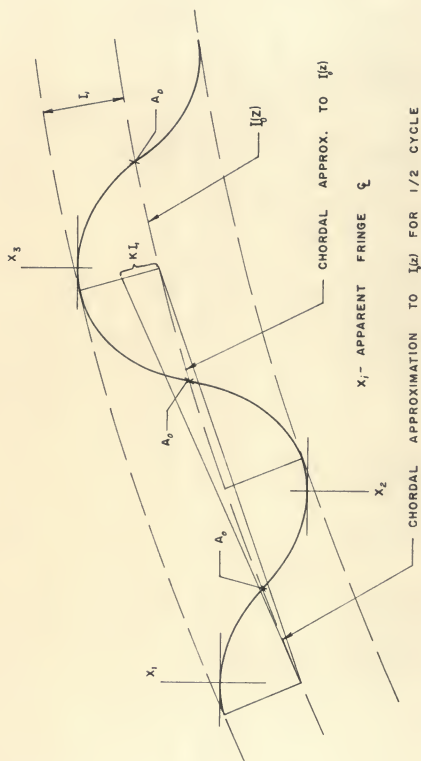


Figure 8. Filtering of Spatial Frequencies Resulting from Heterogeneous Displacements

Figure 9. Showing Definition of K

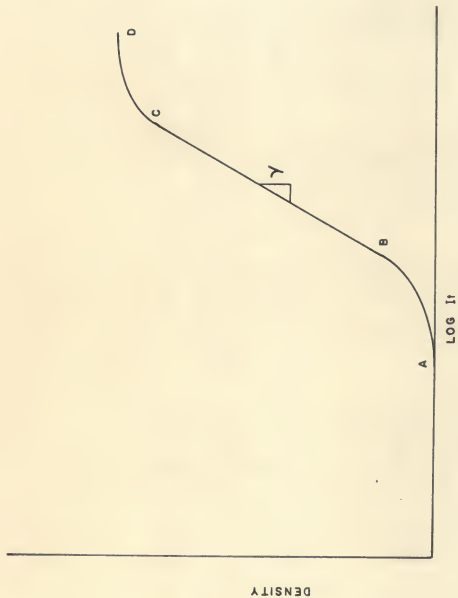
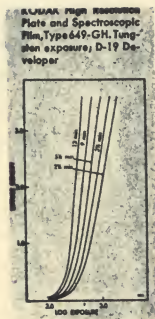
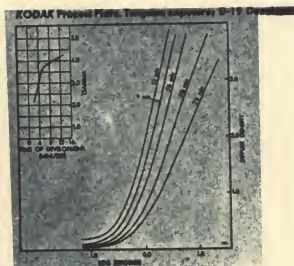


Figure 10. Hurter Driffield Characteristic Curve for Film



CHARACTERISTIC CURVES

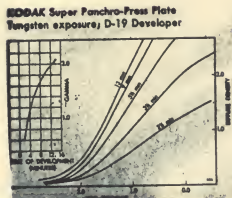
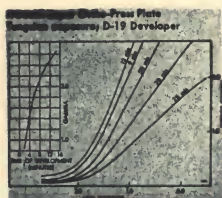


Figure 11. Some Characteristic Curves for Film Used in Moire Work

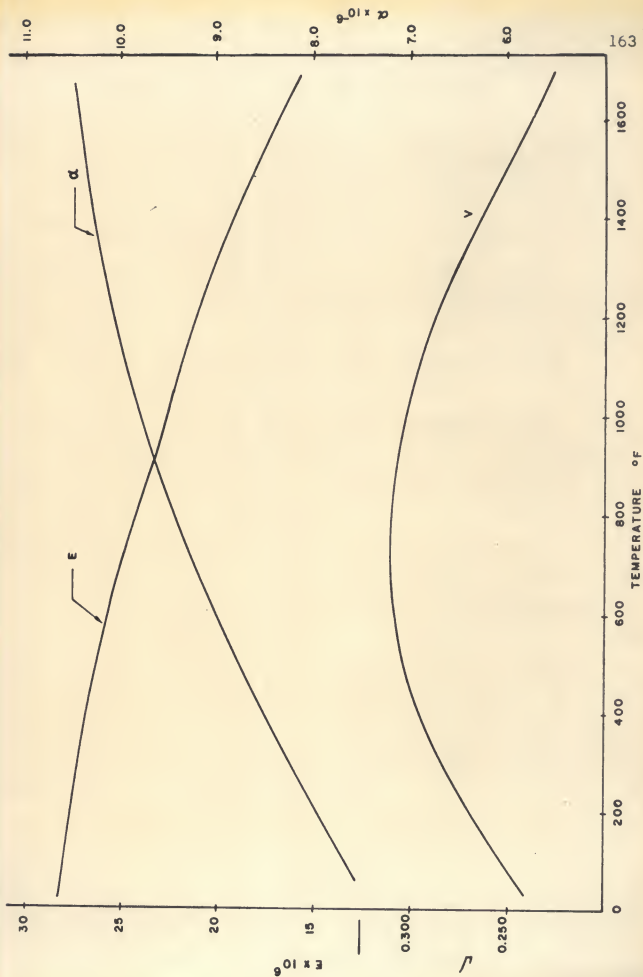


Figure 12. Physical Constants for 304 Stainless Steel



Figure 13. Photo of Modified Enlarger

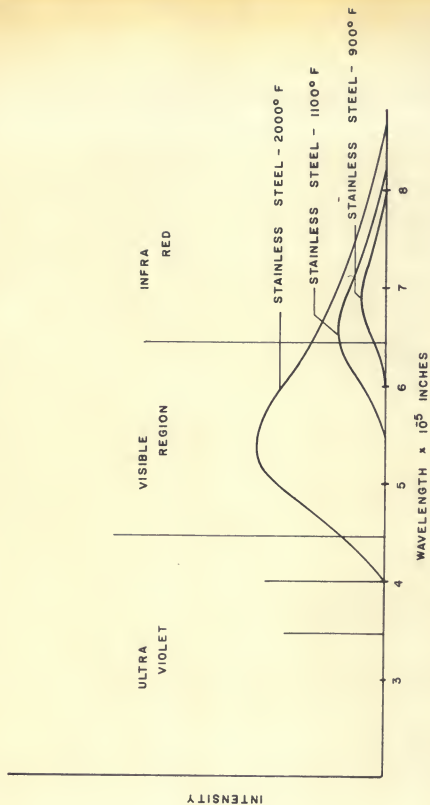


Figure 14. Planck's Law

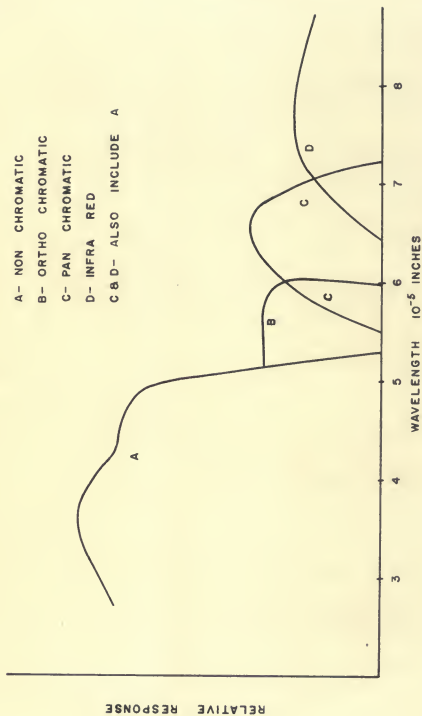


Figure 15. The Spectral Response of Some Films

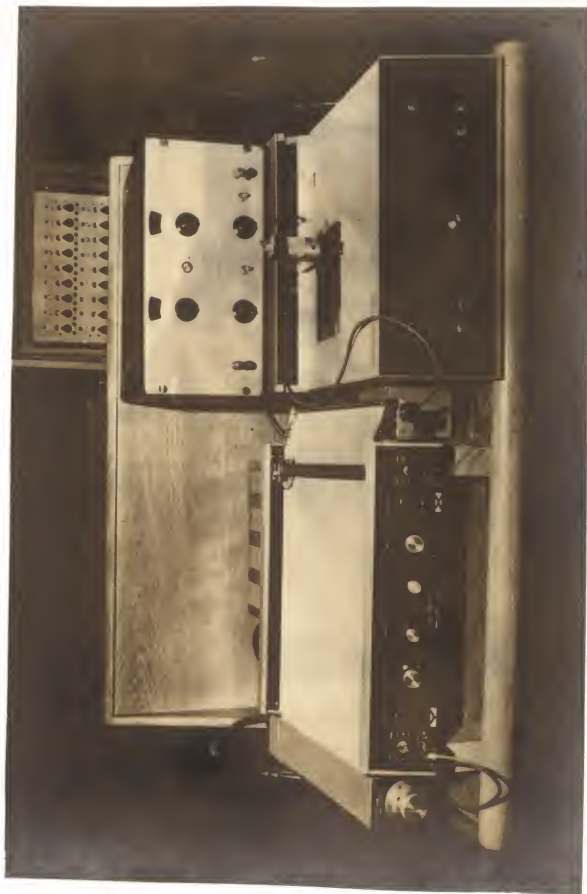


Figure 16. Photograph of Photo-Reader Assembly



Figure 17. Photo-Reader Components

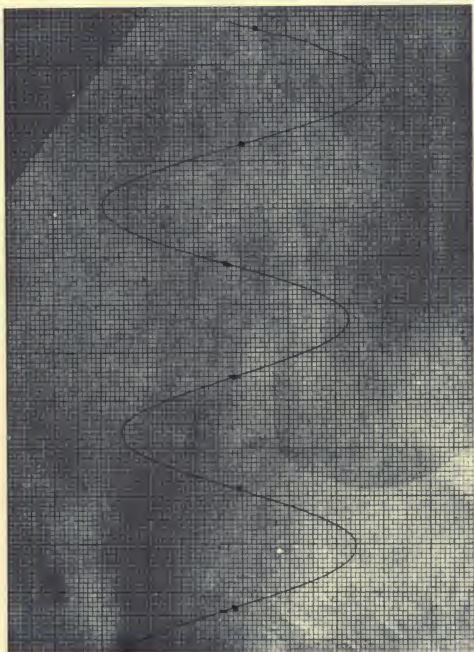


Figure 18. Photo of Photo-reader Curve of Intensity



Figure 19. R.W. 300 - Photo-reader Combination

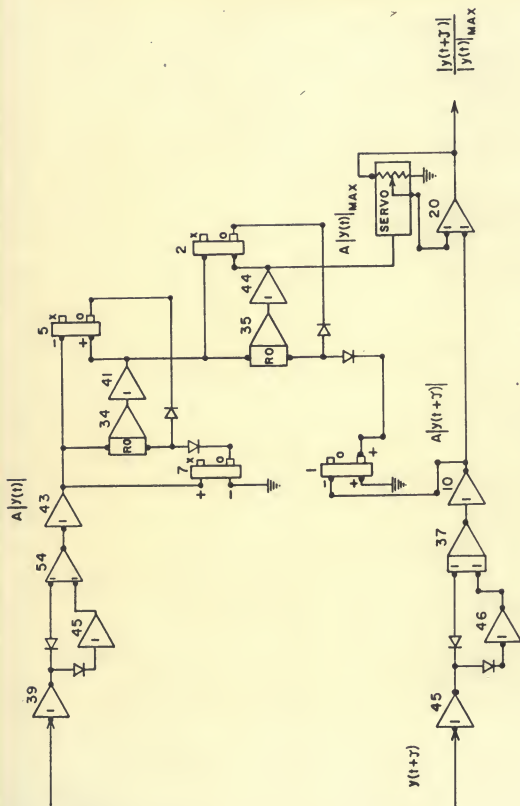


Figure 20. Analog Computer Schematic

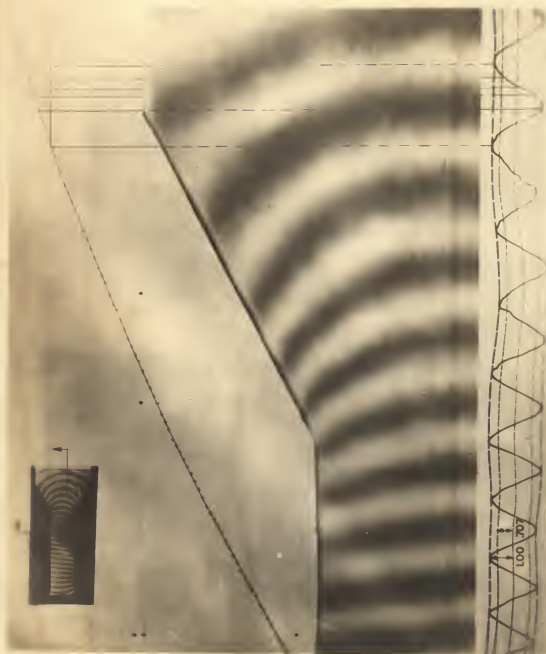


Figure 21. A Tension Test Sample

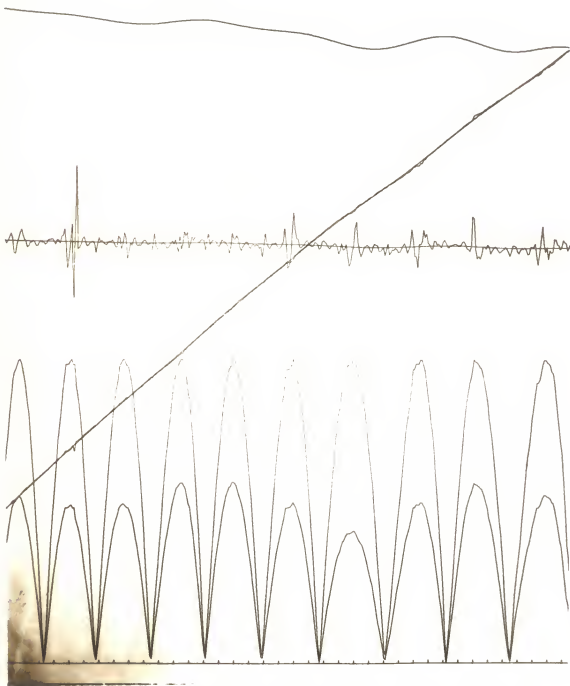


Figure 22. Computer Drawn Curves Corresponding to those of Figure 21



Figure 23. Ring Under Diametral Compression
(a) Initial Mismatch (b) Moire of v
(c) Moire of u

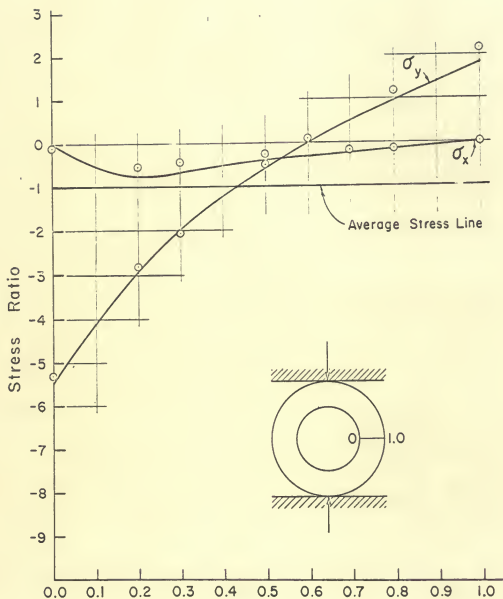


Figure 24. Results of Ring in Compression

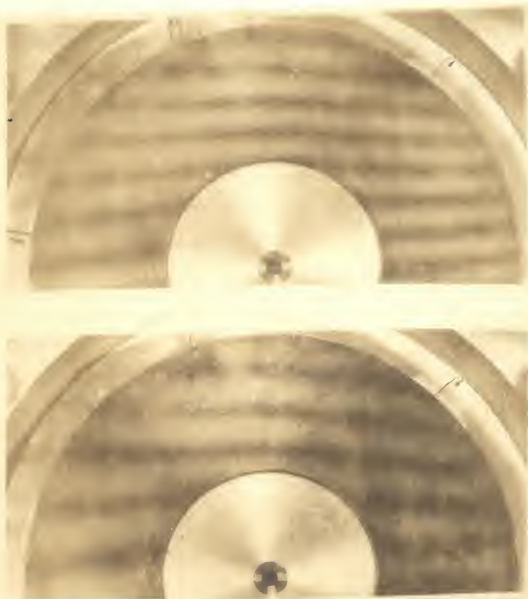


Figure 25. Ring Used in Thermal Test #3--Upper, after Heating
lower, before Heating

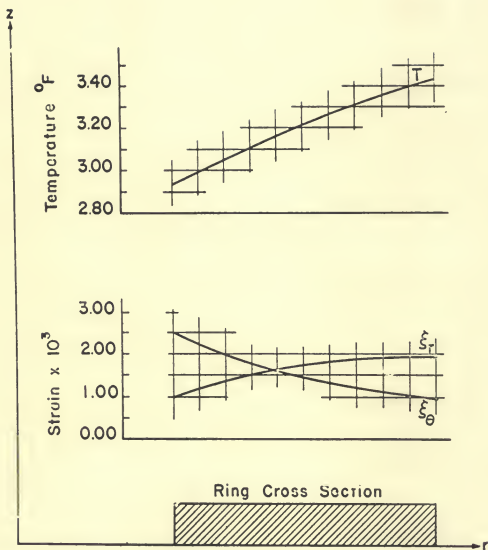


Figure 26. Strain and Temperature Distribution in the Ring Thermal Test

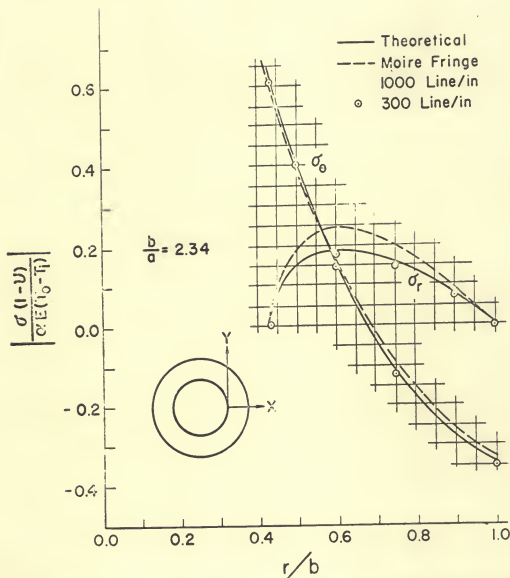


Figure 27. Results of Thermal Test, Comparison between Theoretical Solution, Test Using 1000 Lines/Inch and Discrete Analysis, and 300 Lines/Inch Using Continuous Analysis

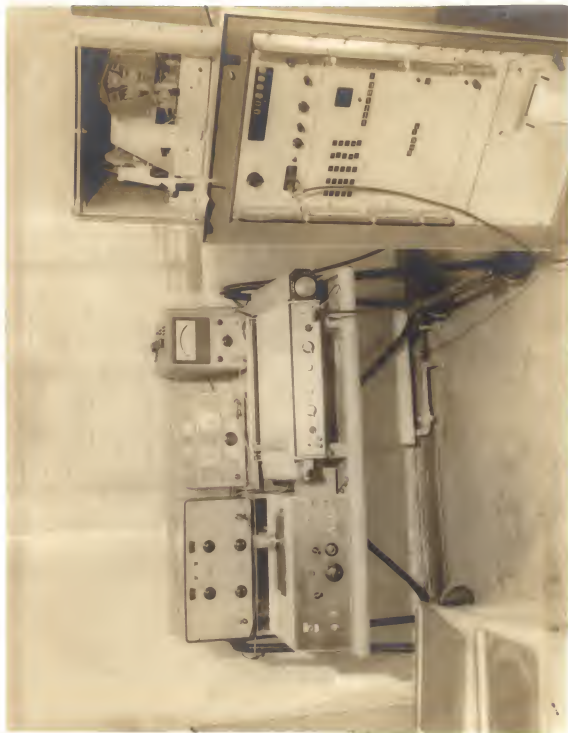


Figure 28. Photo-Reader Dymec 2010B System

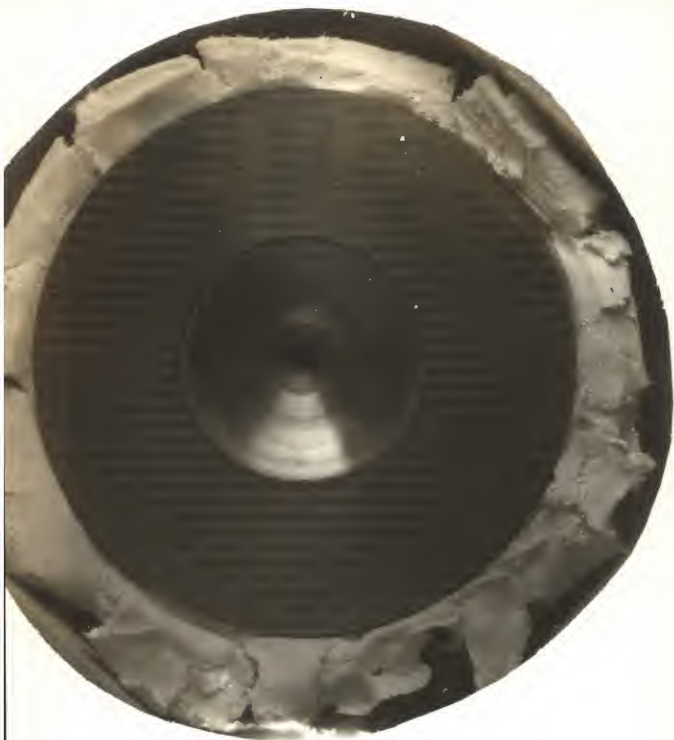


Figure 29. Stainless Steel Ring Thermal Test
before Heating

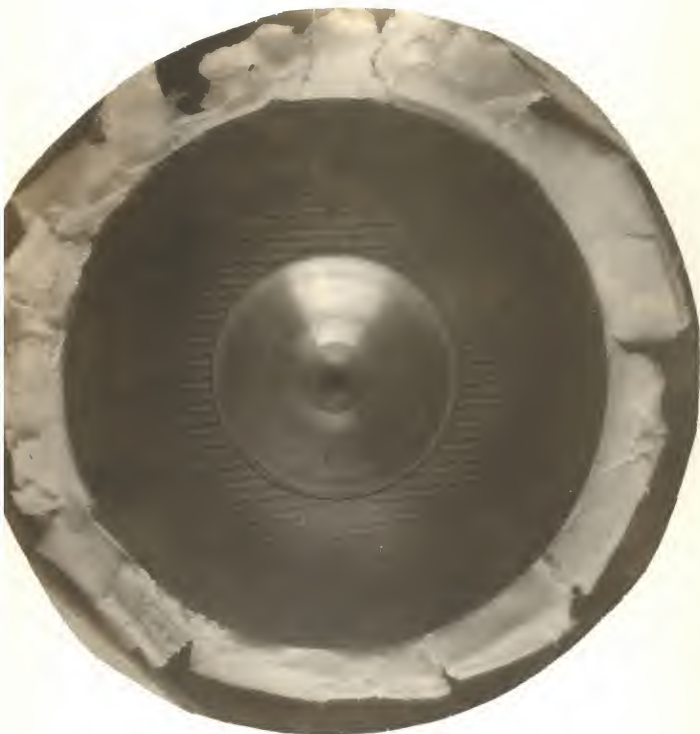


Figure 30. Stainless Steel Ring Thermal Test--
after Heating-- $1,200^{\circ}\text{F}$ O.D.-- 920°F I.D.

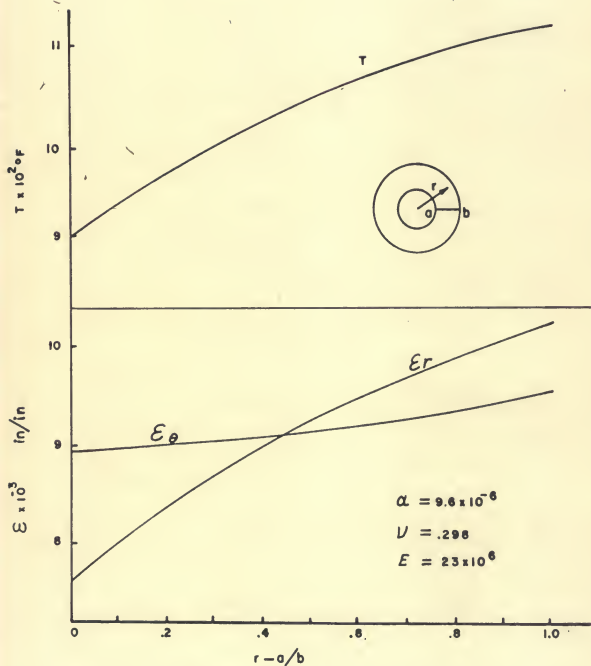


Figure 31. Analytically Obtained Expected Strains

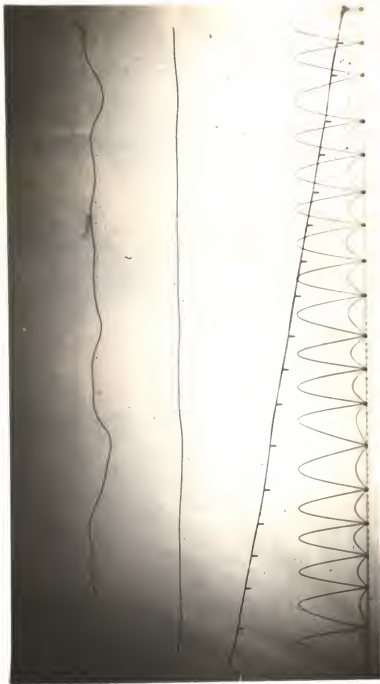


Figure 32. Computer Drawn Curves Ring of Figure 20 before Heating

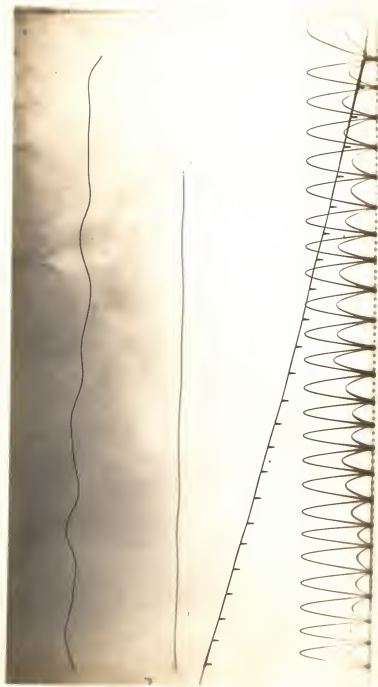


Figure 33. Computer Drawn Curves of Ring of Figure 30 after Heating

[illegible]

Figure 34. Sample Computer Data

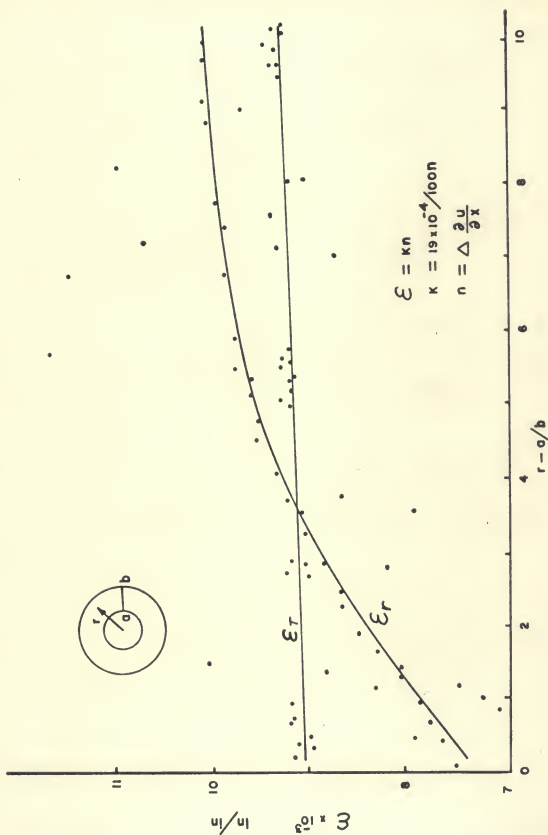


Figure 35. Experimentally Obtained Strains

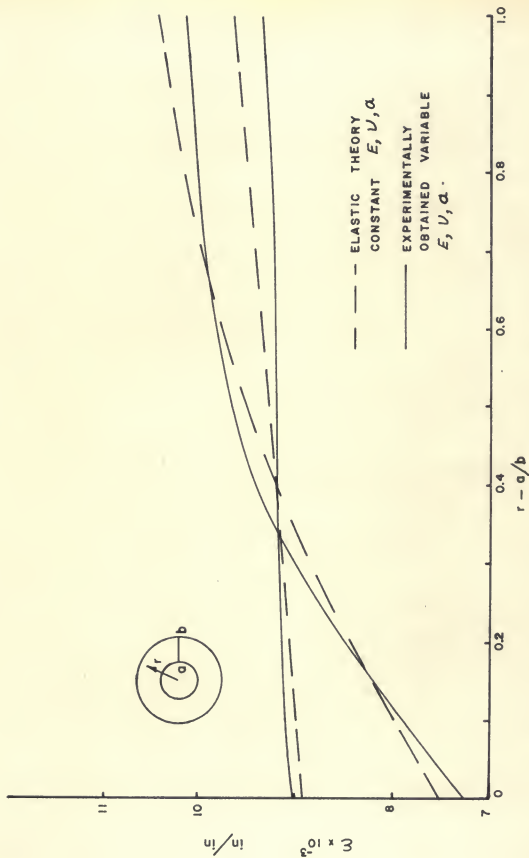


Figure 36. Comparison of Expected and Experimentally Determined Strains

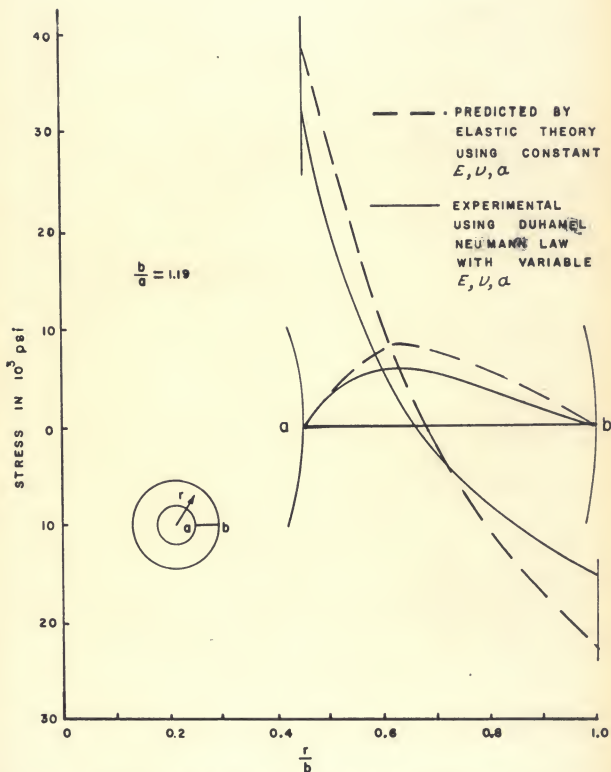


Figure 37. Stress Distribution in a Stainless Steel Ring at $1,120^{\circ}\text{F}$ O.D., and 920°F I.D.

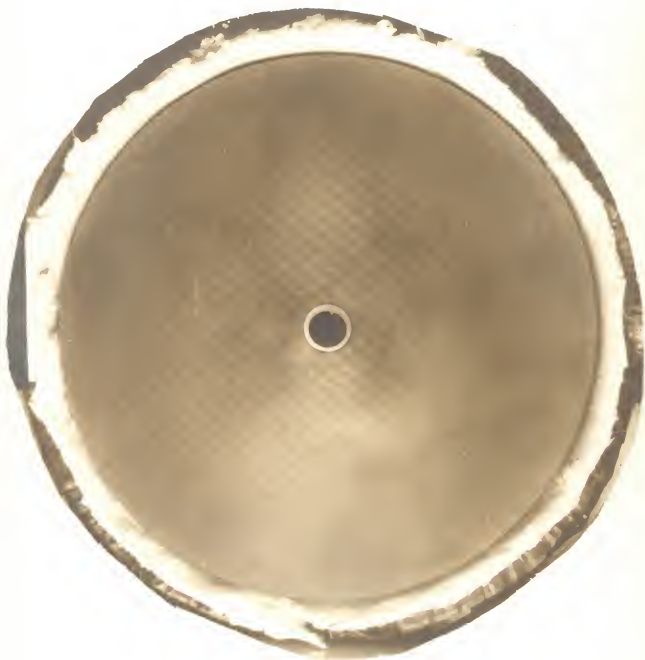


Figure 38. Stainless Steel Ring Number Two--
before Heating



Figure 39. Stainless Steel Ring Number Two Heated to 1,580°F O.D. and 900°F. I.D.

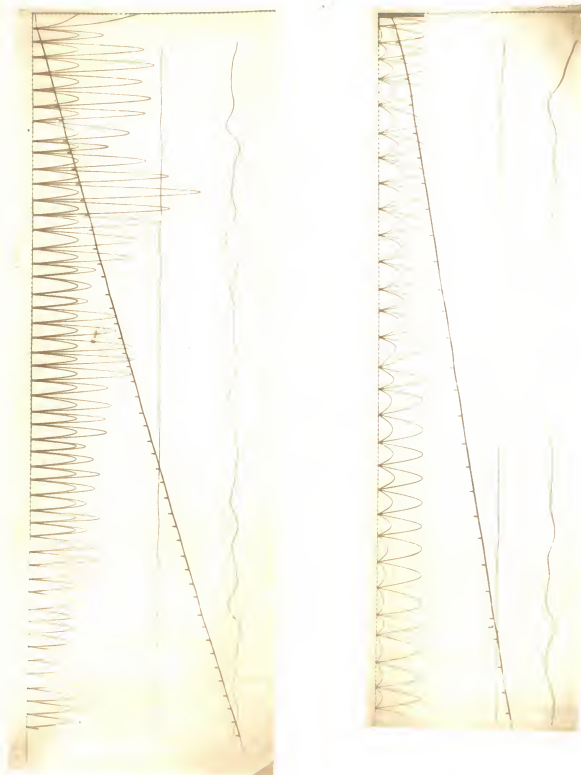


Figure 40. Computer Drawn Curves for the Ring of
Figures 38 and 39

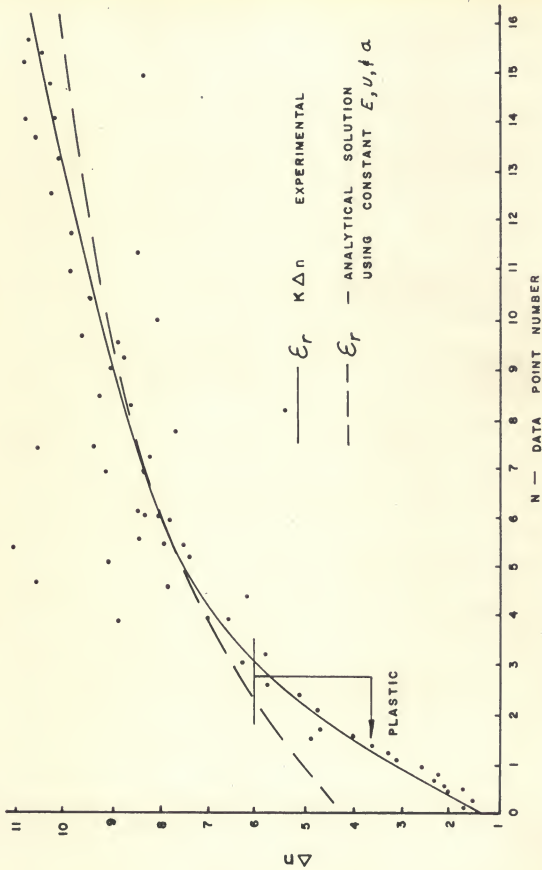


Figure 41. Strains of the Ring of Figure 39 Compared to the Elastic Solution

APPENDIX B

C 1 C RUN 2 FILM 6

```

DIMENSION Z(1700),R(1700),RS(1700),RSS(1700),DR(1700)
DIMENSION DRS(1700),DSRS(1700),TH(1700),BUFFER(200)
DIMENSION Y(1700),YMAX(100),YMIN(100),YMAX(100),NMIN(100)
DIMENSION XNOD1(100),XNOD2(100),XNOD1(100),XNOD2(100),YCOR(1700)
DIMENSION XCOR(1700)

```

```

READ INPUT TAPE 5,100,NDATA,(Y(J),J=1,NDATA)

```

```

WRITE OUTPUT TAPE 6,101,(Y(J),J=1,NDATA)

```

```

IF(Y(2)-Y(1)) 3,8,11

```

```

8 IF(Y(4)-Y(3)) 9,11,11

```

```

9 DO 10 J=1,NDATA

```

```

10 Y(N)=-Y(N)

```

```

11 JJ=0

```

```

N=0

```

```

7 JJ=JJ+1

```

```

12 N=N+1

```

```

ND=N+1

```

```

IF(NDATA-ND) 17,17,29

```

```

29 IF(Y(N+1)-Y(N)) 13,13,12

```

```

13 YMAX(JJ)=Y(N)

```

```

NMAX(JJ)=N

```

```

14 M=N

```

```

15 M=M+1

```

```

ND=N+1

```

```

IF(NDATA-ND) 17,17,28

```

```

28 IF(Y(M+1)-Y(M)) 15,15,16

```

```

16 YMIN(JJ)=Y(M)

```

```

NMIN(JJ)=M

```

```

18 N=M

```

```

GO TO 7

```

```

17 JMAX=JJ-1

```

```

WRITE OUTPUT TAPE 6,102,(YMAX(KJ),NMAX(KJ),YMIN(KJ),NMIN(KJ),KJ=1,
JMAX)

```

2

```

32 DO 30 J=1,JMAX
   YNOD1(J)=(YMAX(J)+YMIN(J))/2.
   K=(NMIN(J)-NMAX(J))/4+NMAX(J)
19 K=K+1
   IF(YNOD1(J)-Y(K)) 19,20,21
20 YNOD1(J)=Y(K)
   XNOD1(J)=K
   GO TO 22
21 YNOD1(J)=Y(K-1)
   XNOD1(J)=K-1
22 YNOD2(J)=(YMAX(J+1)+YMIN(J))/2.
   K=(NMAX(J+1)-NMIN(J))/4+NMIN(J)
23 K=K+1
   IF(Y(K)-YNOD2(J)) 23,24,24
24 YNOD2(J)=Y(K)
30 XNOD2(J)=K
   WRITE OUTPUT TAPE 6,104,(YNOD1(KJ),XNOD1(KJ),YNOD2(KJ),XNOD2(KJ),K
   1J=1,JMAX)
DO 40 J=1,JMAX
SLOPE=(YNOD2(J)-YNOD1(J))/(XNOD2(J)-XNOD1(J))
IF(J-1) 25,25,34
34 IF(J-JMAX) 26,33,33
33 NN=NDATA
GO TO 27

```

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3

25 L=1

NN=XNOD1(1)

DO 45 M=L,NN

X=M-L

XX=XNOD1(1)-X

YLIN=YJOD1(J)-SLOPE*XX

45 YCUR(M)=ABSF(Y(M)-YLIN)

L=XNOD1(1)

NN=XNOD2(1)-1.

DO 47 M=L,NN

X=M-L

YLIN=YJOD1(J)+SLOPE*X

47 YCUR(M)=ABSF(Y(M)-YLIN)

GO TO 46

26 NN=XNOD2(J)-1.

27 L=XNOD1(J)

DO 35 M=L,NN

X=M-L

YLIN=SLOPE*X+YNOD1(J)

35 YCUR(M)=ABSF(Y(M)-YLIN)

IF(J-JMAX) 46,43,43

46 SLOPE=(YNOD1(J+1)-YNOD2(J))/(XJOD1(J+1)-XNOD2(J))

KK=XNOD1(J+1)-1.

NN=XNOD2(J)

DO 40 M=NN,KK

X=M-NN

YLIN=SLOPE*X+YNOD2(J)

40 YCUR(M)=ABSF(Y(M)-YLIN)

43 CONTINUE

DO 90 JA=1,187

90 XCUR(JA)=JA

WRITE OUTPUT TAPE 6,201

100 FORMAT(14/6(2X,F7.3,2X))

101 FORMAT(1H1 5H DATA / (2X,10F11.3))

4 102 FORMAT(//7X, 4H MAX 23X,4H MIN// (2X,F10.4,15,10X,F10.4,15))
 104 FORMAT(// 6H NODES / (10F11.3))

C THIS IS THE COUPLING POINT BETWEEN THE NORMALIZING PROGRAM AND
 C PLUTTING PROGRAM.

N=NDATA

K=0

NI=0

CALL PLOTS (BUFFER(200),200)

WRITE OUTPUT TAPE 6,201

J = 1

DO 81 I=1,N

81 TH(I+1) = TH(I) + .05

2 K = K+1

IF(YCOR(K+1)-YCOR(K)) 3,2,2

3 YM=YCOR(K)

4 K = K+1

IF(YCOR(K)-YCOR(K+1))5,4,4

5 DO 60 I=J,K

Z(I)=YCOR(I)/YM

G=YCOR(I)-YCOR(I-1)

H = SQRT(1.-Z(I)**2)

H = SIGNF (H,G)

5

IF(H) 37,88,87

88 D = 1.5729

GO TO 99

87 D = ATANF(Z(I)/H)

89 IF(D) 32,83,83

82 D = D+3.14159

83 R(I) = D+A

NI = NI+1

IF (NI-N) 60,6,6

60 CONTINUE

A = A+3.14159

J = K+1

GO TO 2

6 CONTINUE

AA = 41.

NA = N-1

NC = N-2

DO 51 I=2,NA

51 DR(I) = R(I+1)-R(I-1)

IA = N-20

DO 64 IB=21,1A

QA = 0.

ID = IB-20

IE = IB+20

DO 65 IC=ID,IE

65 QA = QA+R(IC)

64 RS(IB) = QA/AA

IA = 1A-20

NB = 1A-41

DO 69 IB=41,1A

QA = 0.

6

```

10 = 10-20
1E = 10+20
DO 70 IC=10,1E
70 QA = QA+RS(1,IC)
69 RSS(1B) = QA/AA

```

```

1A = 1A-1
DO 66 I=43,1A
66 DRS(I) = RSS(I+1)-RSS(I-1)

```

```

1A = 1A-20
ND = 1A-63
DO 67 IB=63,1A
QA = 0.
10 = 10-20
1E = 10+20
DO 68 IC=10,1E

```

```

68 QA = QA+DRS(1,IC)
67 DRS(1,IC) = QA/AA

```

```

WRITE OUTPUT TAPE 6,21,( 1, YCOR(1), Z(1), R(1), RS(1), RSS(1), DR(1), DR
1S(1), DRS(1), I=1, NDATA)

```

XXX=NDATA

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7
 AXIS=XXX/40.

NE = N-70

NF = NC-70

CALL PLOT (3.,2.,-3)

CALL AXISB (0.,0.,26.,90.,.5)

CALL AXISB (0.,0.,80.,0.,.5)

CALL LINE (TH,YCOR,NE,1,1)

CALL LINEB (TH,2,NE,1,1,1,5.)

CALL LINEB (TH,R,NE,1,1,1,15)

CALL LINEB (TH(41),RSS(41),NB,1,1,1,15)

CALL PLOT (0.,13.,-3)

CALL LINEB (TH(63),DSRS(63),ND,1,1,1,5.)

CALL PLOT (0.,3.,-3)

CALL LINEB (TH(63),DSRS(63),ND,1,1,1,30.)

CALL EXIT

201 FORMAT(1H1 ///)

91 FORMAT(1X,14,8E14.4)

END(1,0,0,0,0,0,0,0,0,0,0,0,0,0)

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BIOGRAPHICAL SKETCH

Bernard E. Ross was born November 15, 1919, at Princeton, Minnesota. In June, 1937, he was graduated from Columbia Heights High School in Columbia Heights, Minnesota. From 1941 to 1945 he served with the United States Air Force in Europe. Following his discharge in 1945, he enrolled in the School of Engineering at Vanderbilt University. He received a Bachelor of Mechanical Engineering degree in 1948. In September, 1948, he enrolled in the Graduate School of the Chrysler Institute of Engineering and was awarded a Master of Automotive Engineering degree in 1950. He joined the faculty of the School of Engineering at Vanderbilt University in 1952. In 1958 he attained the rank of Associate Professor of Engineering Mechanics. He obtained a leave of absence in September, 1961, and enrolled in the Graduate School of the University of Florida to work toward the degree of Master of Science in Engineering, which he received in December, 1962.

Mr. Ross is married to the former Vivian Nancy Morris and is the father of four children. He is a member of the American Rocket Society, American Society of Automotive Engineers, American Society of Mechanical Engineers, and Tau Beta Pi.

J. R. Elwell
J. Williams